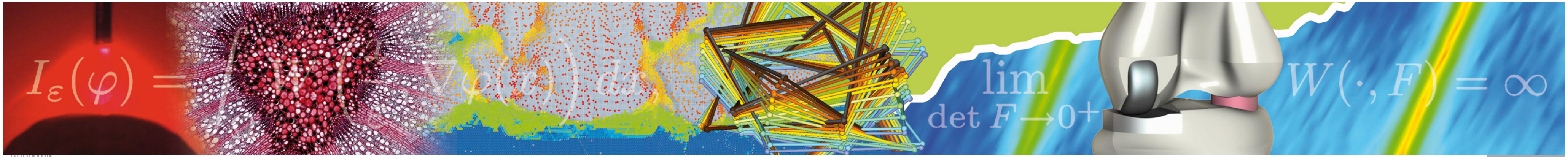


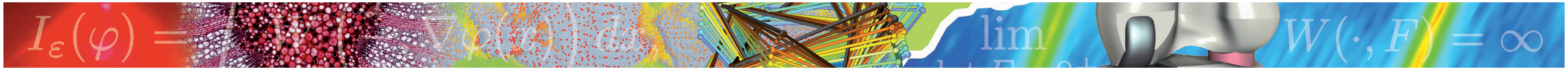
Thermodynamique des Processus Irréversibles
Quiberon, 16-22 sept. 2018
7ème école d'été de mécanique théorique

Effets de couplage et effets dissipatifs accompagnant la déformation des matériaux solides

(3ème partie)

André Chrysochoos
LMGC, UMR 5508 CNRS-UM

LABORATOIRE DE MÉCANIQUE ET GÉNIE CIVIL - UM/CNRS



Programme

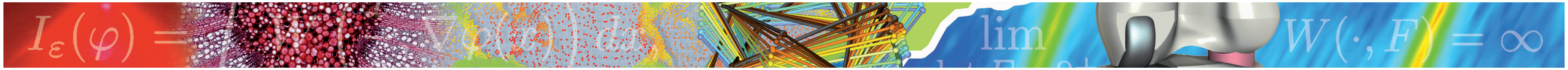
1 - Cadre thermomécanique et bilan d’énergie
(TPI-MSG)

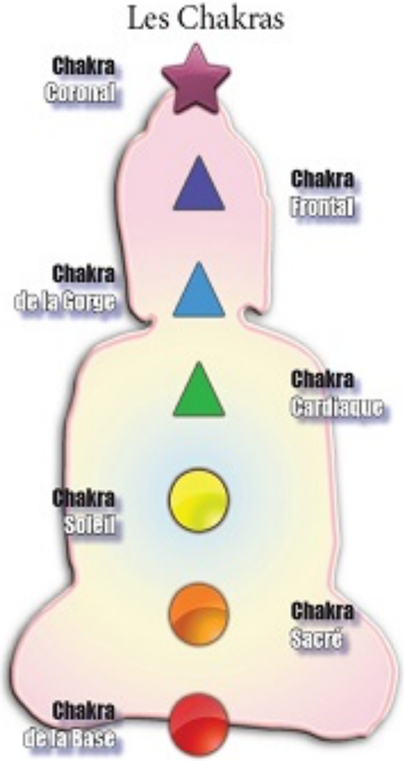
2 – Quelques éléments rhéologiques à la sauce MSG
(d’une vision mécanique à vision thermomécanique)

3 - Analyse expérimentale des bilans d’énergie
(imagerie quantitative)

4 - Effet du temps : couplage thm et/ou viscosité ?
(interaction forte et/ou irréversibilité)

5 - Effet dissipatif dans les métaux
(fatigue : HCF & VHCF)





BILAN ENERGETIQUE - GRATUIT

Laissez-vous surprendre par la pertinence de ce bilan offert et découvrez comment :

- ☆ *Avoir plus d'énergie !*
- ☆ *Augmenter votre taux vibratoire !*
- ☆ *Vivre en pleine forme !*

Bilan d'énergie :
le retour !

Energy balance : reminder (I)

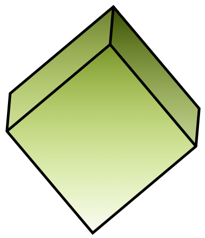
$$w^{\bullet}_{def} = \sigma : \dot{\varepsilon} = \sigma^{r} : \dot{\varepsilon} + \sigma^{ir} : \dot{\varepsilon}$$

$$= \underbrace{\sigma^{r} : \dot{\varepsilon} + A_{\alpha}.\dot{\alpha}}_{w^{\bullet}_{e} + w^{\bullet}_{s}} + d_1$$

$(...)^{\bullet}$ path-dependence

$w^{\bullet}_{e}$: rate of elastic energy

d_1 : intrinsic dissipation



$w^{\bullet}_{s}$: rate of strored energy

... incomplete balance !!

Energy balance : reminder (II)

- rate of internal energy

$$\rho\dot{e} = \rho C\dot{T} + \underbrace{(\sigma^r : \dot{\varepsilon} + A.\dot{\alpha})}_{\dot{w}_e + \dot{w}_s} - \underbrace{(T\sigma^r_{,T} : \dot{\varepsilon} + TA_{,T}.\dot{\alpha})}_{\dot{w}_{thc}}$$

$$= \rho C\dot{T} + \dot{w}_e + \dot{w}_s - \dot{w}_{thc}$$

« thc » = thermomechanical couplings

- heat equation

$$\rho C\dot{T} + \mathrm{div}\,\mathbf{q} = \underbrace{\boldsymbol{\sigma}^{ir} : \dot{\varepsilon} - \mathbf{A}.\dot{\alpha}}_{d_1} + T\boldsymbol{\sigma}^r_{,T} : \dot{\varepsilon} + T\mathbf{A}_{,T}.\dot{\alpha} + r_e$$

- comments

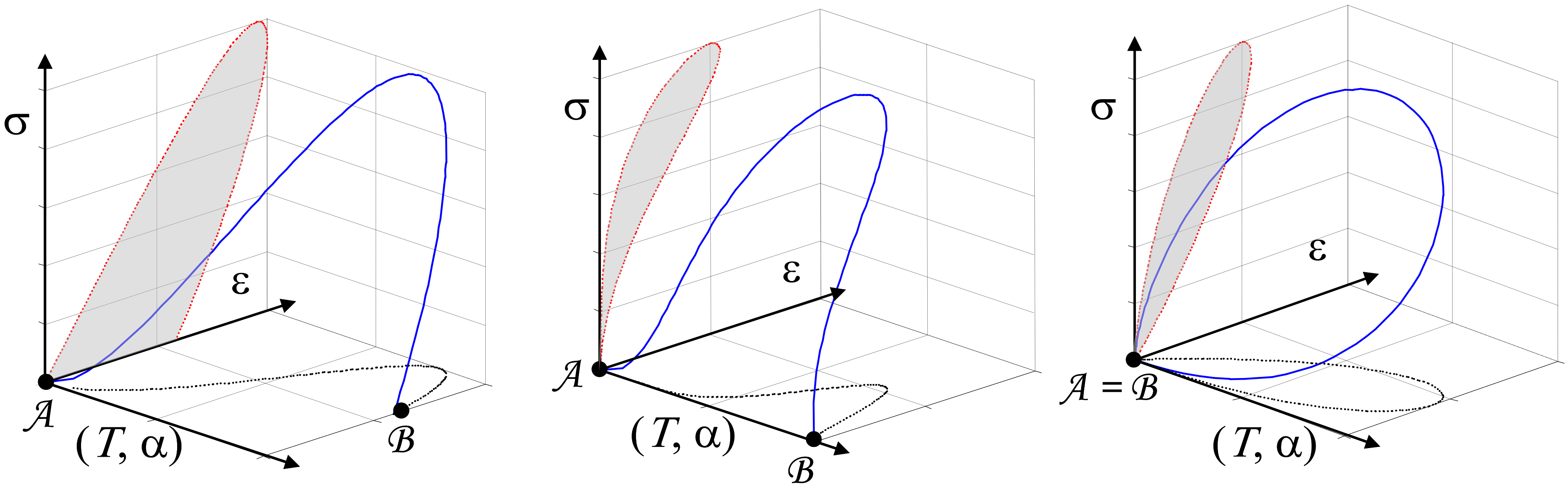
C.1: C specific heat

C.2: $q = -k.gradT$

⚠ kinematics required

C.3: $\dot{T} = \dfrac{\partial T}{\partial t} + v.\nabla T$

Focus on a load-unload cycle



(i) $\mathcal{A} \neq \mathcal{B}$ *(ii)* $\varepsilon_{\mathcal{A}} = \varepsilon_{\mathcal{B}}$ *(iii)* $\mathcal{A} = \mathcal{B}$

(i) $$w_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} \sigma : \dot{\varepsilon}\, dt = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1\, dt + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} (\rho \dot{e} - \rho C \dot{T} + w_{\text{thc}}^{\bullet})\, dt$$

(ii) Hysteresis loop : $w_{\text{def}} = A_{\text{h}}$ (for uniaxial loading)

(iii) Load-unload cycle = thermodynamic cycle $$w_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1\, dt + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} w_{\text{thc}}^{\bullet}\, dt$$

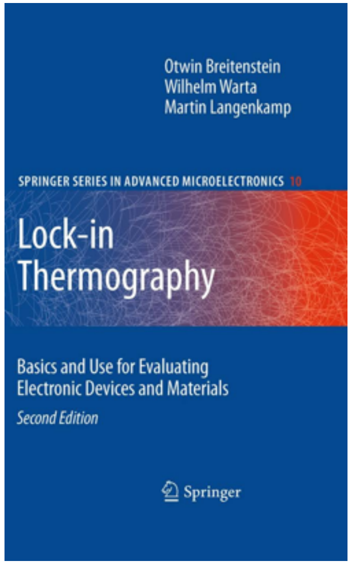
– 4 –

Effet du temps : couplage thm et/ou viscosité ?

(interaction forte et/ou irréversibilité)

Thermoélasticité standard

TSA : analyse thermique des contraintes



Modèle thermoélastique linéaire isotrope homogène

- mécanique : mémoire sélective
- thermodynamique : pas de dissipation intrinsèque
- variables d'état : (θ, ε), $\theta = T - T_0$ (HPPT $\theta \ll T_0$)

Physique 1 : Superposition des déformations élastiques et thermiques

Version 1 D

$$\varepsilon = \frac{\sigma^r}{E} + \alpha_{th}\theta$$

$$\sigma^r = E(\varepsilon - \alpha_{th}\theta)$$

Version 3 D

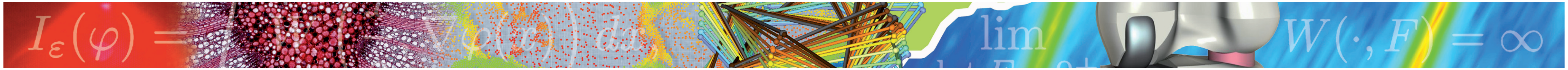
$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma^r_{ij} - \frac{\nu}{E}\sigma^r_I\delta_{ij} + \alpha_{th}\theta\delta_{ij}$$

$$\sigma^r_{ij} = \lambda\varepsilon_I\delta_{ij} + 2\mu\,\varepsilon_{ij} - (3\lambda + 2\mu)\theta\delta_{ij}$$

Physique 2 : chaleur spécifique constante

$$C_\varepsilon = -T\frac{\partial^2\psi}{\partial T^2} \approx -T_0\frac{\partial^2\psi}{\partial T^2} = C_0 \quad \Rightarrow \quad \frac{\partial^2\psi}{\partial T^2} = -\frac{C_0}{T_0}$$

α_{th} dilatation
C_ε chal. spé.
$\varepsilon_I = tr(\varepsilon)$
$\varepsilon_{II} = ½\, \varepsilon_{ij}\varepsilon_{ij}$



$$\chi = \frac{E\alpha_{th}^2 T_0}{\rho_0 C_0}$$

- énergie libre

1D
$$\rho_0 \psi(\theta, \varepsilon) = \frac{1}{2} E(\varepsilon - \alpha_{th}\theta)^2 - \frac{\rho_0 C_0}{2T_0}(1+\chi)\theta^2$$

3D
$$\rho_0 \psi(\theta, \underline{\underline{\varepsilon}}) = \frac{1}{2}\left(\lambda \varepsilon_I^2 + 4\mu\, \varepsilon_{II}\right) - (3\lambda + 2\mu)\alpha_{th}\theta\varepsilon_I - \frac{\rho_0 C_0 \theta^2}{2T_0}$$

- équations d'état

$$\sigma_{ij}^r = \rho \frac{\partial \psi}{\partial \varepsilon_{ij}} = \lambda \varepsilon_I \delta_{ij} + 2\mu\, \varepsilon_{ij} - (3\lambda + 2\mu)\alpha_{th}\theta\delta_{ij}$$

$$s = -\frac{\partial \psi}{\partial \theta} = \frac{1}{\rho}(3\lambda + 2\mu)\alpha_{th}\varepsilon_I + \frac{\rho_0 C_0 \theta}{T_0}$$

- potentiel de dissipation et équations complémentaires

$$\phi(\vec{q}, \underline{\underline{\dot{\varepsilon}}}) = \frac{k^{-1}\vec{q}.\vec{q}}{2T_0}$$

k tenseur de conduction

$$\underline{\underline{\sigma}}^{ir} = \frac{\partial \phi}{\partial \underline{\underline{\dot{\varepsilon}}}} = \underline{\underline{0}}$$

$$\vec{q} = -k\,\overrightarrow{\text{grad}}\,\theta$$ *Fourier*

Aspects thermiques et calorifiques

Equation générale de diffusion

$$\rho_0 C_0 \dot{T} + \mathrm{div}\,\mathbf{q} = \underline{\underline{\boldsymbol{\sigma}}}^{ir} : \underline{\underline{\dot{\boldsymbol{\varepsilon}}}} - \mathbf{A}.\dot{\alpha} + T\,\underline{\underline{\boldsymbol{\sigma}}}^{r}_{,T} : \underline{\underline{\dot{\boldsymbol{\varepsilon}}}} + T\,\mathbf{A}_{,T}.\dot{\alpha} + r_e$$

$A_i = \partial\psi / \partial\alpha_i$

Variables internes

Matériau thermoélastique isotrope homogène

$$\rho_0 C_0 \dot{T} - k\Delta T = \cancel{\underline{\underline{\boldsymbol{\sigma}}}^{ir}} : \underline{\underline{\dot{\boldsymbol{\varepsilon}}}} - \cancel{\mathbf{A}}.\dot{\alpha} + T\,\underline{\underline{\boldsymbol{\sigma}}}^{r}_{,T} : \underline{\underline{\dot{\boldsymbol{\varepsilon}}}} + T\,\cancel{\mathbf{A}_{,T}}.\dot{\alpha} + r_e$$

Source thermoélastique

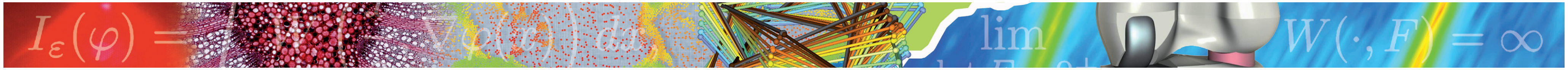
$$s_{the} = T\,\underline{\underline{\boldsymbol{\sigma}}}^{r}_{,T} : \underline{\underline{\dot{\boldsymbol{\varepsilon}}}} = T\frac{\partial 2\psi}{\partial T\,\partial\varepsilon_{ij}}\dot{\varepsilon}_{ij} = -(3\lambda + 2\mu)T\alpha_{th}tr(\dot{\varepsilon}_{ij}) = -3KT\alpha_{th}\dot{\varepsilon}_I$$

Modèle simple (i.e. différentiel) de diffusion [AC, Photomécanique, 95]

$\theta = T - T_0$ (HPPT $\theta \ll T_0$), sources homogènes, fuites linéaires, …

la solution spectrale limitée au 1er vecteur propre…

$$\dot{\theta} + \frac{\theta}{\tau_{th}} = -\frac{3KT\alpha_{th}}{\rho_0 C_0}\dot{\varepsilon}_I$$



❑ Application à l'analyse thermique des contraintes (TSA)

P. Stanley & J. Barton (UK), J. Rowlands (AUS), U. Galietti (IT), …
Dispositif SPATE (79), Deltatherm (96) …

$$s_{the} = -3KT\alpha_{th}\dot{\varepsilon}_I = -T\alpha_{th}\dot{\sigma}_I - 9K\alpha_{th}^2\dot{\theta}$$

$$\left(1 + \underbrace{\frac{9K\alpha^2 T_0}{\rho_0 C_0}}_{<<1}\right)\dot{\theta} - \underbrace{\frac{k}{\rho_0 C_0}}_{\text{diffusivité}}\Delta\theta = -\frac{\alpha_{th}T}{\rho_0 C_0}\dot{\sigma}_I$$

Si processus adiabatique et faibles variations de température …

$$\delta\theta = -\frac{T_0\alpha_{th}}{\rho_0 C_0}\delta\sigma_I = K_{TSA}\,\delta\sigma_I$$

« Vous mesurez des températures, vous mesurez des contraintes ! »

TSA : un exemple d'application industrielle

Pierre Bremond
Agema – Additionnal
Cedip – Flir

New developments in Thermo Elastic Stress Analysis by Infrared Thermography.

P. Bremond – Cedip IR systems

IV Conferencia Panamericana de END Buenos Aires – 2007

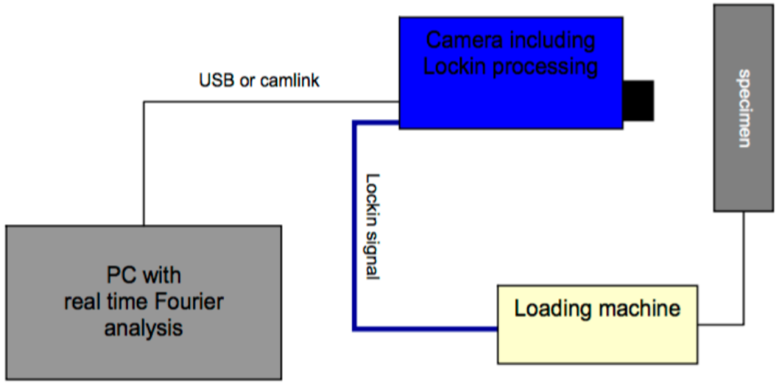


Figure 1. Signal processing into Thermographic Stress Analyzer

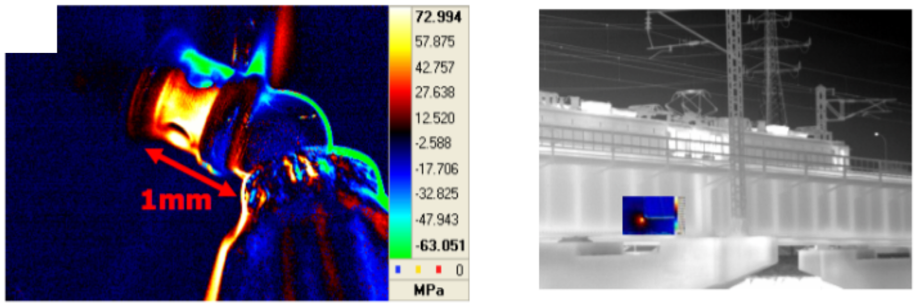


Figure 4. Size of object analyzed by TheSA

TheSA systems have a large panel of applications. Basically the use of imaging measurement systems is required when localization of high stress is unknown or to compare to another full field data like FEA results.

R. Williams

SFT
GDR 2519
PM
SEM

TSA : bilan qualitatif

Points forts :

- Méthode « simple » à mettre en œuvre
- Champ du premier invariant du tenseur des contraintes
- Contraintes planes :

  TSA + Photoélasticimétrie ➔ contraintes principales

  Cf. [Barone, Wang, Patterson, IUTAM 1998]
- Concentration de contraintes sur structures planes, biréfringentes

Limitations :

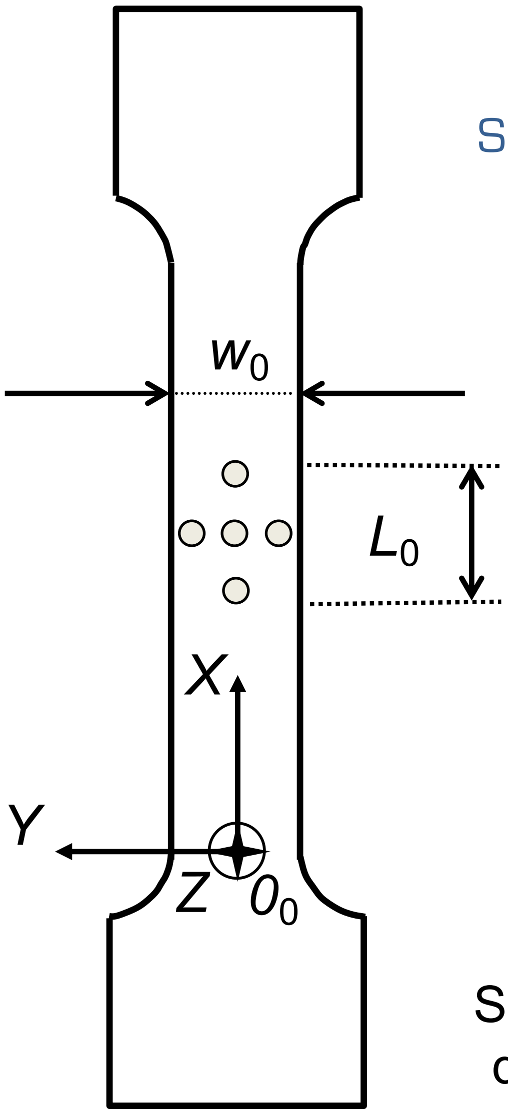
- Mesure de θ : pièces massives complexes, corps lambertiens,
- Diffusion, rayonnement parasites… stabilité thermique…
- Adiabaticité
- Comportement élastique … caractéristiques homogènes, constantes …

$$\delta\theta = -\frac{T_0 \alpha_{th}}{\rho_0 C_0} \delta\sigma_I$$

Elasticité caoutchoutique

Inversion thermoélastique

Rubber elasticity [Caborgan et al., 2009]



Specimen and material

gauge part

L_0 = 5 mm

w_0 = 6 mm

t_0 = 2.1 mm

Specimen: thin, flat dog-bone shaped

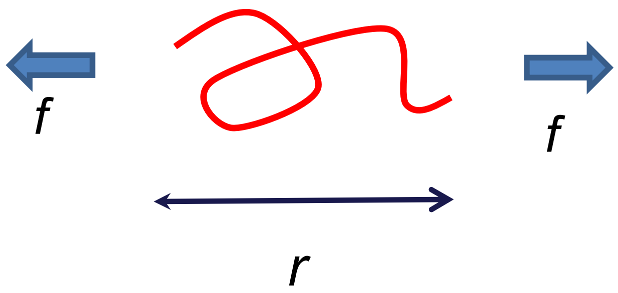
« ***natural* » *rubber (NR)***

cis-1,4 polyisoprene,
per 100 g 3 g stearic acid,
9.85 g zinc oxide,
2 g antioxidant,
3 g sulphur,
3 g plasticiser,
4 g accelerators

heated at 160 °C for 10 minutes,
stored at - 10 °C, in dry air

Rubber elasticity: molecular approach (I)

Molecular chain



$$f = \frac{3kT}{\langle r_0^2 \rangle} r$$

Entropic elasticity

$$f = \left(\frac{\partial \Psi}{\partial r}\right)_T = \cancel{\left(\frac{\partial E}{\partial r}\right)_T} - T\left(\frac{\partial S}{\partial r}\right)_T$$

$$S = k \log(\Omega)$$

Ω: number of conformations

k: Boltzmann constant

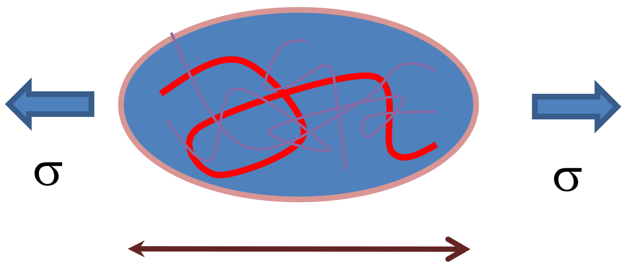
$\langle r_0^2 \rangle$ m. s. end-to-end distance of the chain

- f vs. r: linear $\Rightarrow$ « elasticity »
- f vs. T: F ↗ with T !! $\Rightarrow$ different from standard thermoelasticity where F ↘ with T + volume variation (i.e. thermodilatability) !!

Rubber elasticity: molecular approach (II)

Chain → Network (N_{ch} chains)

Statistical modeling of molecular network



$$\lambda = \sqrt{\frac{\langle r^2 \rangle}{\langle r_0^2 \rangle}}$$

- incompressibility assumption
- uniaxial loading in the direction 1

- affine model [Flory, 1953]
- isotropic material

$$\Delta\psi_{nwk} = -T\Delta S_{nwk} = \frac{N_{ch}kT}{2}(I_1 - 3)$$

$$I_1^2 = \mathrm{tr}(F^T F) = \sum_{i=1}^{i=3} \lambda_i^2$$

$F = \dfrac{\partial x}{\partial X}$ transformation gradient tensor

λ_i extension ratios

tensile Cauchy stress

$$\sigma_{11} = N_{ch}kT(\lambda_1^2 - \lambda_1^{-1}) = N_{ch}kT(e^{2\varepsilon_{11}} - e^{-\varepsilon_{11}})$$

with ε the Hencky strain tensor

Rubber elasticity: mechanical approach (III)

Hyperelasticity : continuum thermomechanical approach

- non linear elasticity at finite strain
- objectivity (strain invariants)
- initial isotropy
- isochoric transformation
- often isothermal models

A lot of « phenomenological » models, e.g. Mooney-Rivlin (1948), [Ogden, ...]

$$\psi(\lambda_i) = C_1(I_1 - 3) + C_2(I_2 - 3) \quad \xrightarrow{\text{tension}} \quad \sigma_{11} = 2(\lambda_1^2 - \lambda_1^{-1})(C_1 + C_2\lambda_1^{-1})$$

with $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \quad I_3 = \lambda_1^2\lambda_2^2\lambda_3^2 = 1$

Rubber elasticity: modified entropic elasticity (III)

(i) Entropic effects : Gough (1805) – Joule (1857)

$$e(s,\varepsilon) = e_c(T) \quad \text{analogy of perfect gaz} \quad \psi_c(T,\varepsilon) = TK_1(\varepsilon) + K_2(T)$$

(ii) Inversion of thermoelastic effect : Anthony (1942)

standard thermoelasticity at low extension ratio, rubber effects at high strain ratio

(i), (ii) literature: modified entropic elasticity

$$e(s,\varepsilon) = e_1(T) + e_2(\varepsilon)$$ [Chadwick and Creasy, 1984]

drawback : strongly non linear expression,
many parameters hardly identifiable

Rubber behavior: a simple euristic model (IV)

Proposal:

- « leave » the classical framework of (entropic) elasticity
- interpret the « thermoelastic inversion » as a competition between 2 coupling mechanisms
- describe this competition introducing a « rubber strain tensor » ε_c in a series model

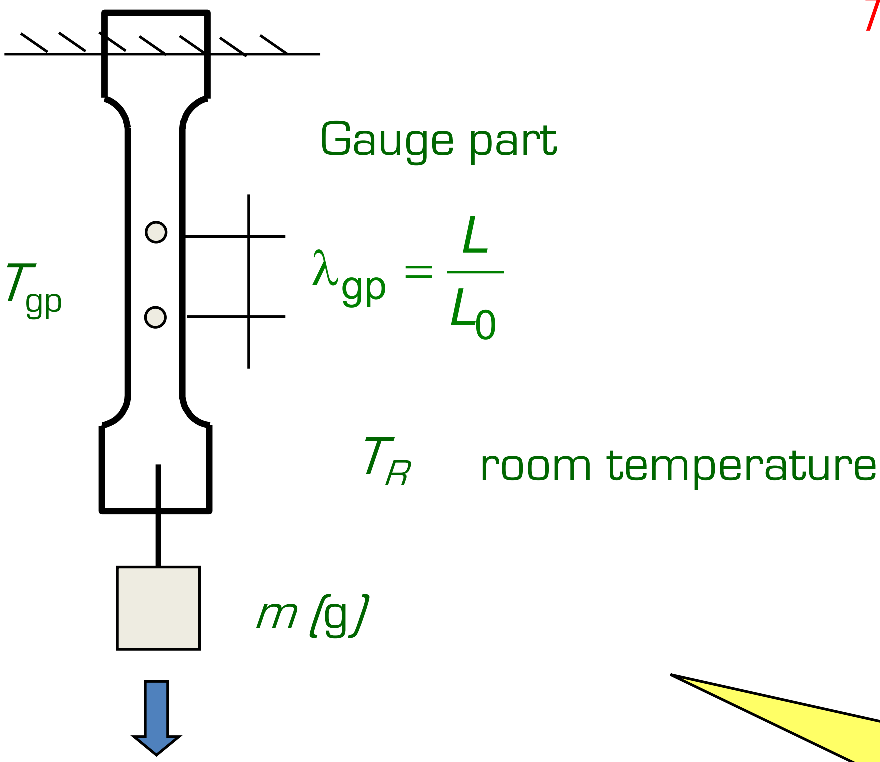
$$\psi(T,\varepsilon,\varepsilon_c) = \psi_{the}(T,\varepsilon-\varepsilon_c) + \psi_c(T,\varepsilon_c)$$

standard thermoelastic free energy — free energy of affine model

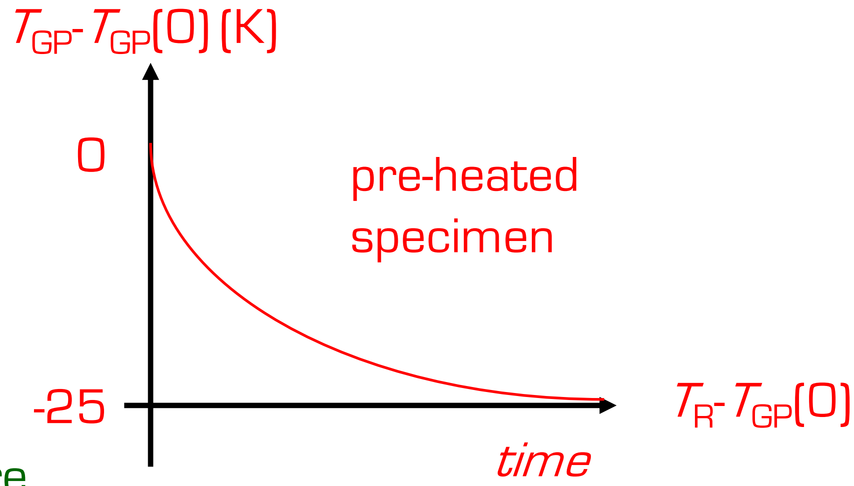
a first step … towards possible irreversibility ; Mullins effects [Mullins, 1948]

$$\varphi(q,\dot{\varepsilon},\dot{\varepsilon}_c;T) = \frac{q^2}{2kT} + \eta\dot{\varepsilon}_c^2$$

Thermoelastic inversion : protocol



Mechanical loading
(constant weight)



Thermal loading
(thermal return)

Output signal : longitudinal strain

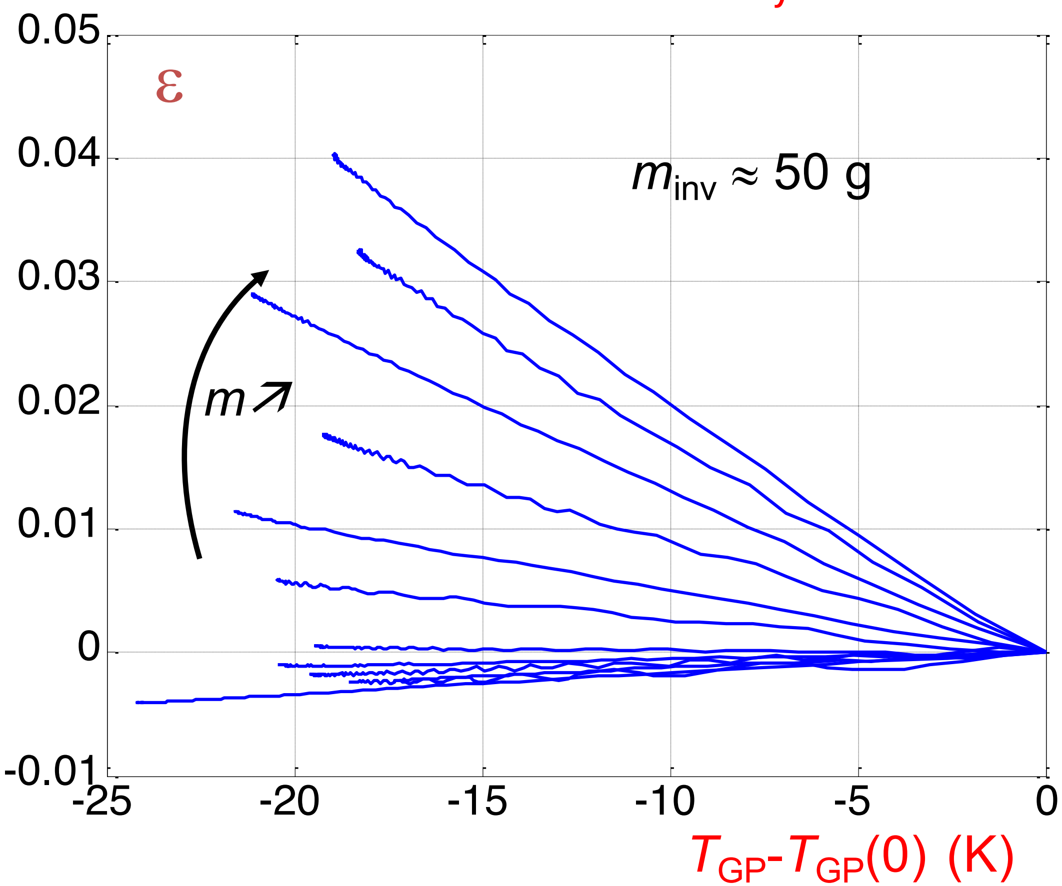
$\varepsilon = \log \lambda_{gp}$

Thermoelastic inversion : results

Anthony , 1942: relaxation test

No viscosity here !?

PHD Caborgan 2010 : creep test



$$\varepsilon \leq 0 \text{ if } m \leq m_{\text{inv}}$$

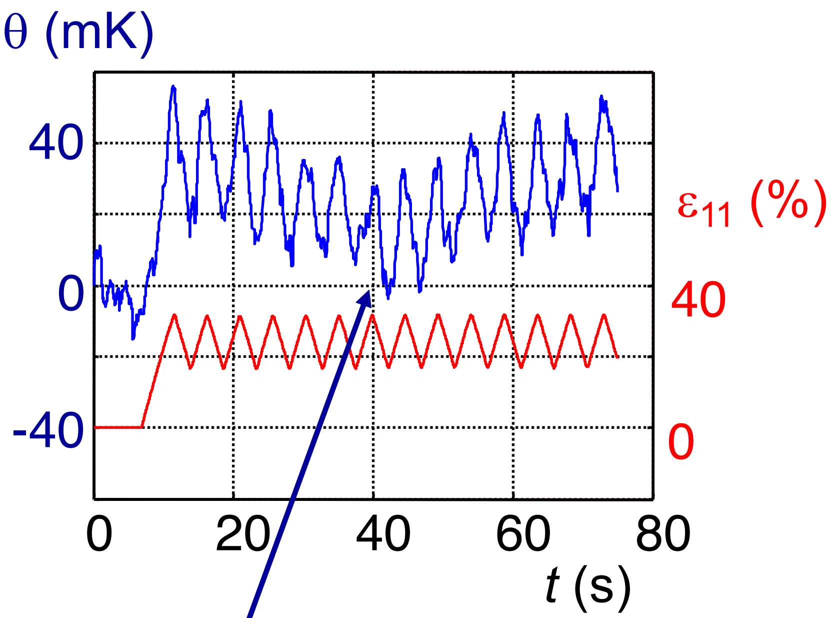
Predominance of thermoelastic coupling

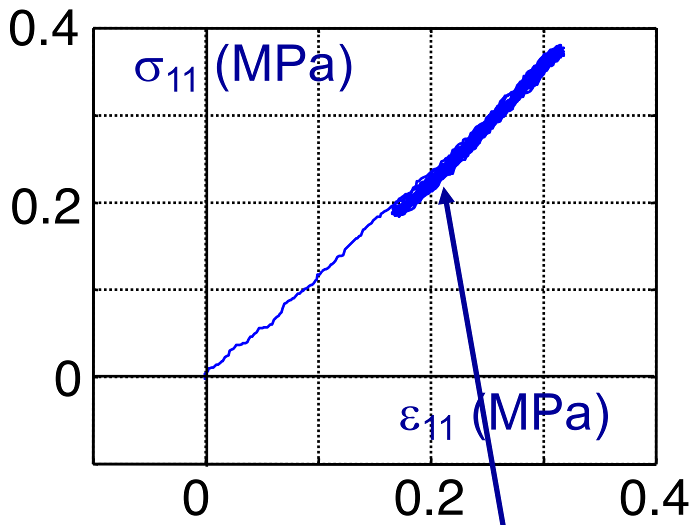
$$\varepsilon > 0 \text{ if } m > m_{\text{inv}}$$

Predominance of rubber coupling

m = 0 3 30 40 50 60 100 200 300 400 520 640 730

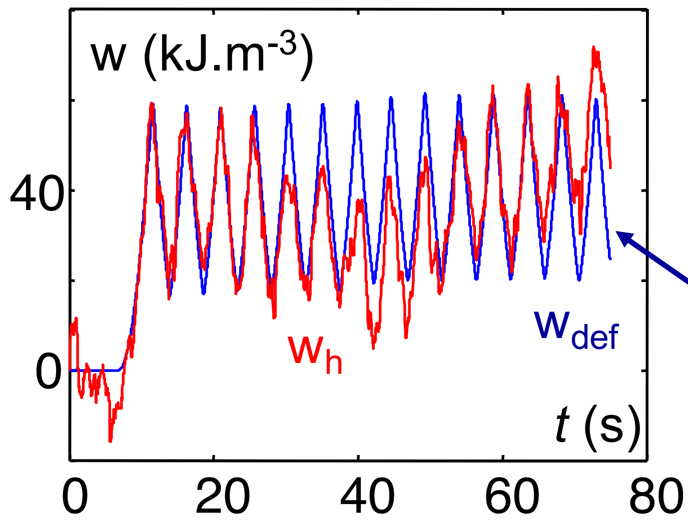
Load-unload cycles : experimental results





Oscillation
Drift (±10 mK)

« Quasi-elastic »
response



$w_{def} \approx w_h$

$e \approx e_c[T]$

Euristic 1D model: identification

volume free energy

$$\psi(\theta,\varepsilon,\varepsilon_c)=\underbrace{\frac{E}{2}\left(\varepsilon-\varepsilon_c-\alpha_{th}\theta\right)^2-\left(\frac{\rho C}{T_0}+E\alpha_{th}^2\right)\frac{\theta^2}{2}}_{\psi_{the}}+\overbrace{K_c(T_0+\theta)\left(\frac{e^{2\varepsilon_c}}{2}+e^{-\varepsilon_c}-\frac{3}{2}\right)}^{\psi_c}$$

dissipation potential

$$\varphi(q,\varepsilon,\varepsilon_c)=\frac{q^2}{2kT_0}+\eta\,\dot{\varepsilon}_c^2$$

material constants

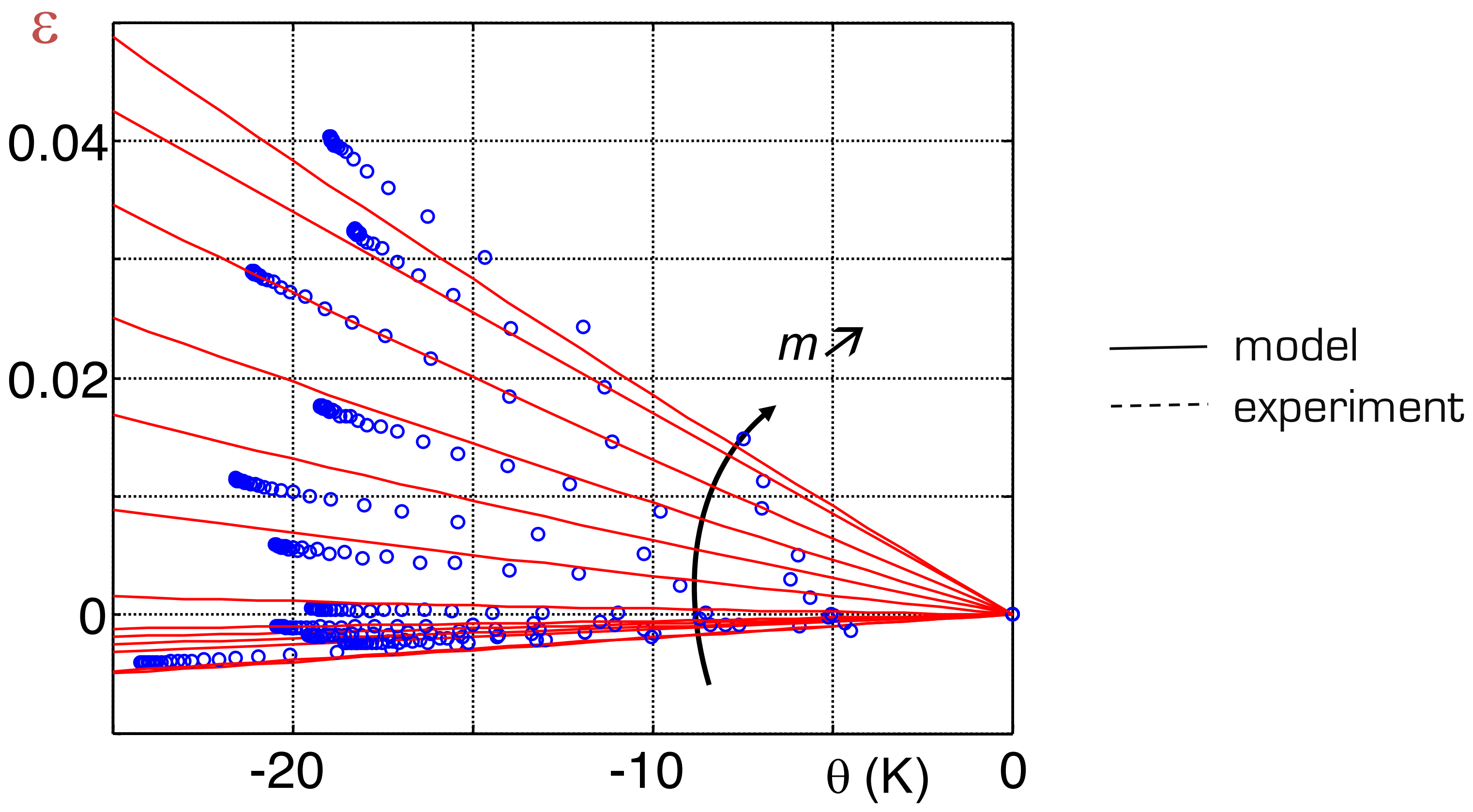
ρ = 950 kg.m^{-3}, C = 1150 J.kg^{-1}.K^{-1}, τ_{eq} = 20 s (0D diffusion model)

E = 100 MPa , α_{th} =2.10^{-4} K^{-1}

K_c = 1110 Pa.K^{-1}

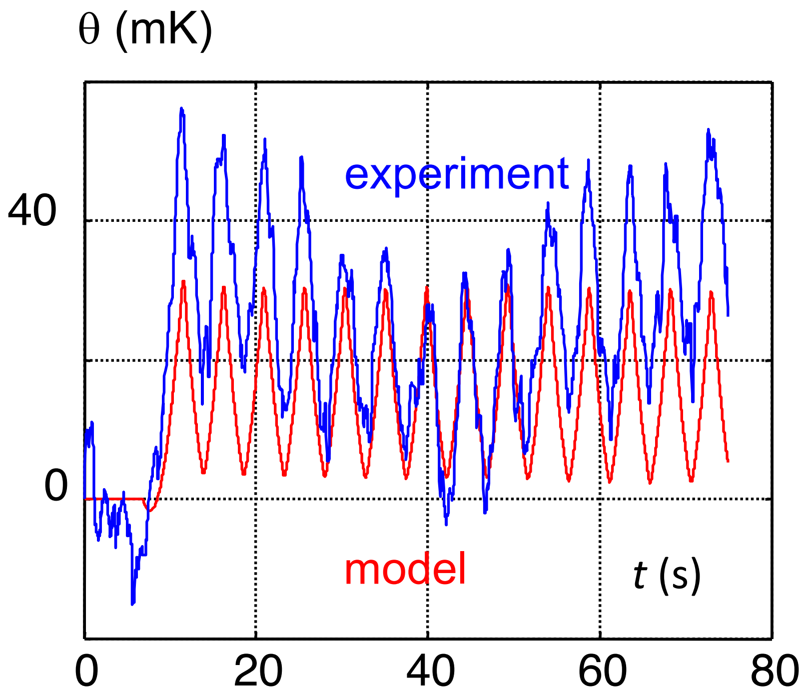
η = 0 MPa.s, no viscous effect…

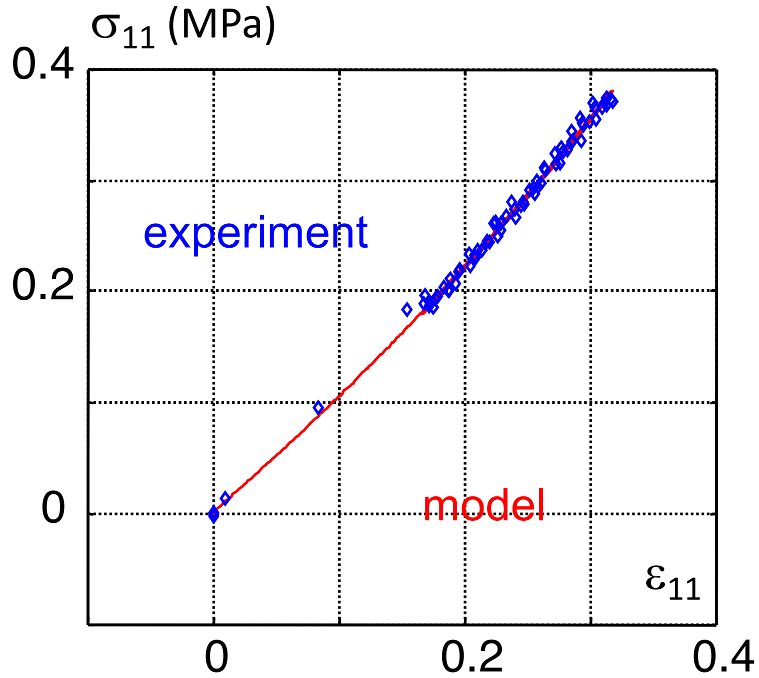
Thermoelastic inversion : exp. vs. num. responses



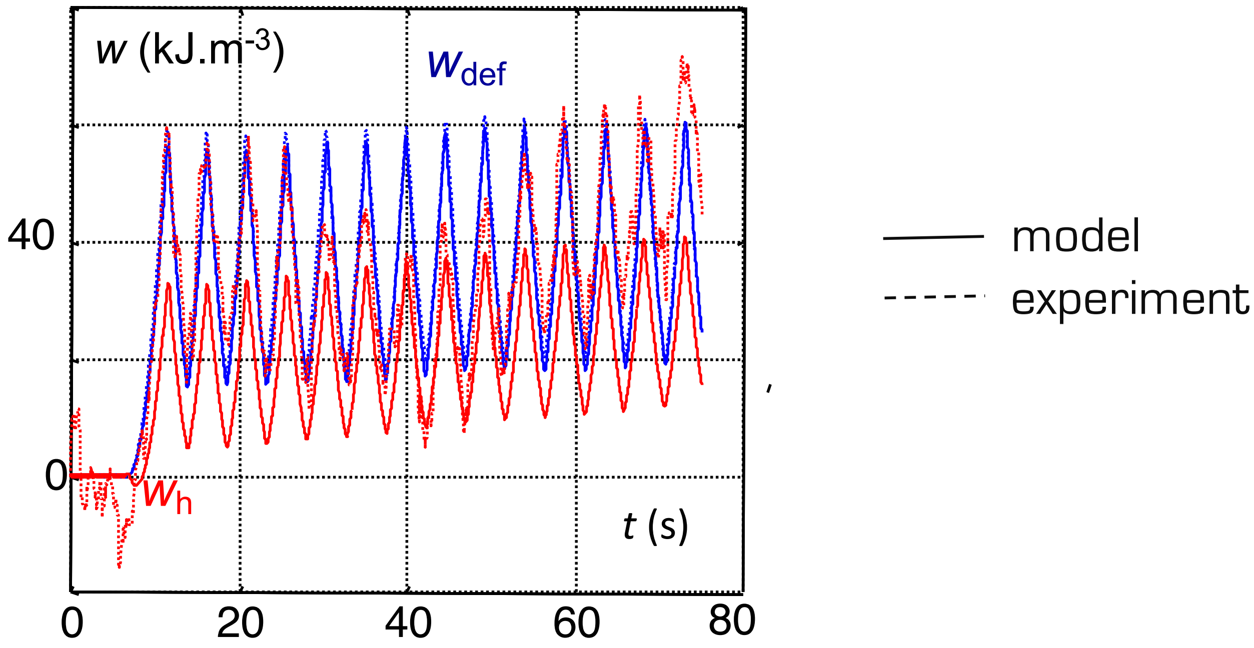
m = 0 3 30 40 50 60 100 200 300 400 520 640 730

Load-unload cycles : exp. vs. num. responses

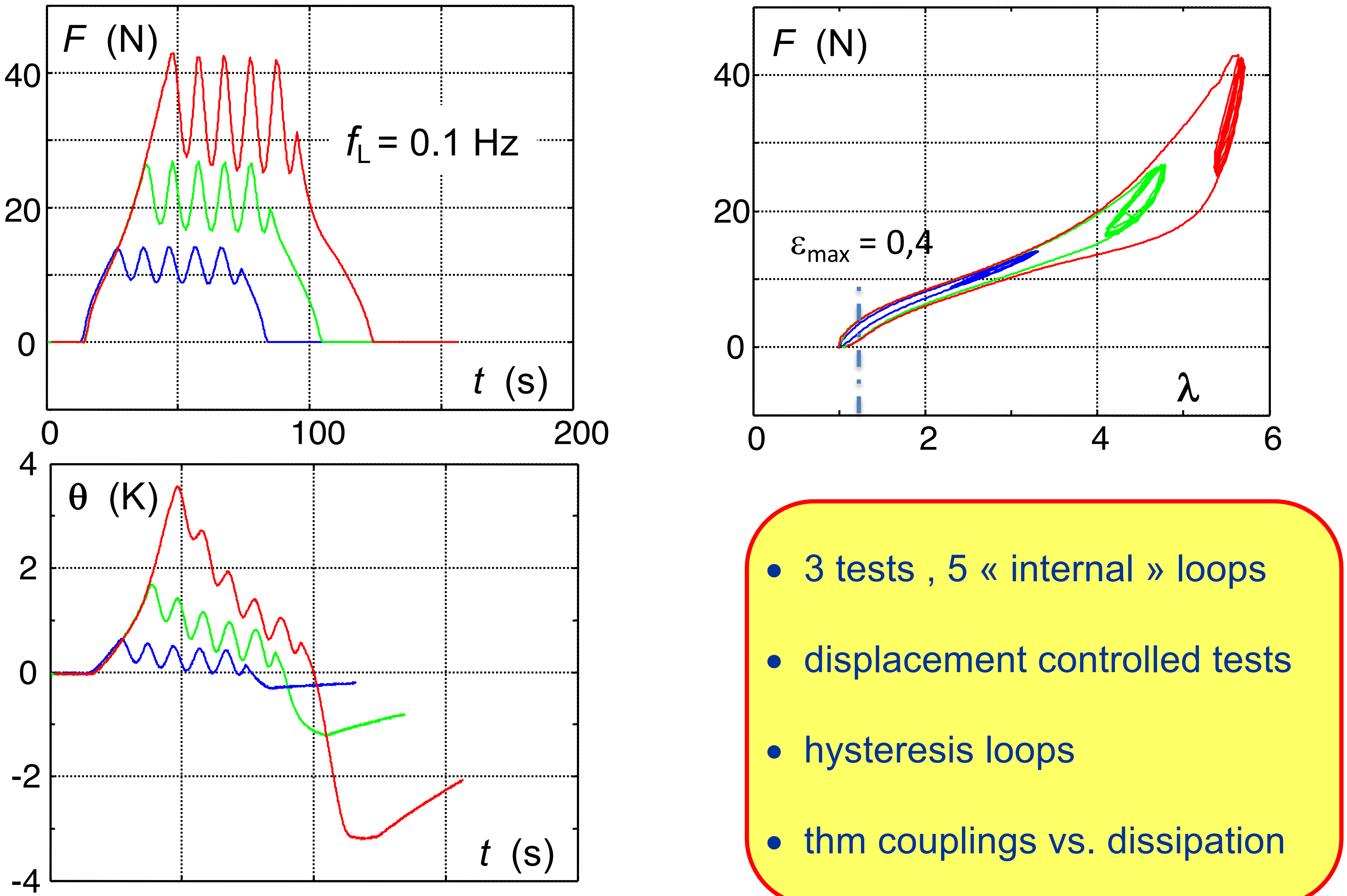




ε_{max} = 0,4



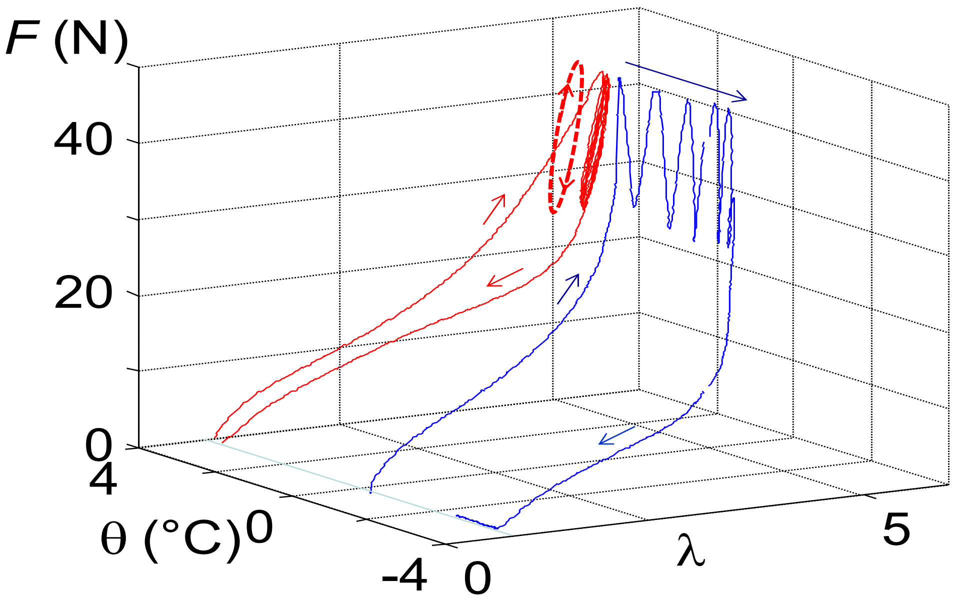
Load- unload test (large strain)

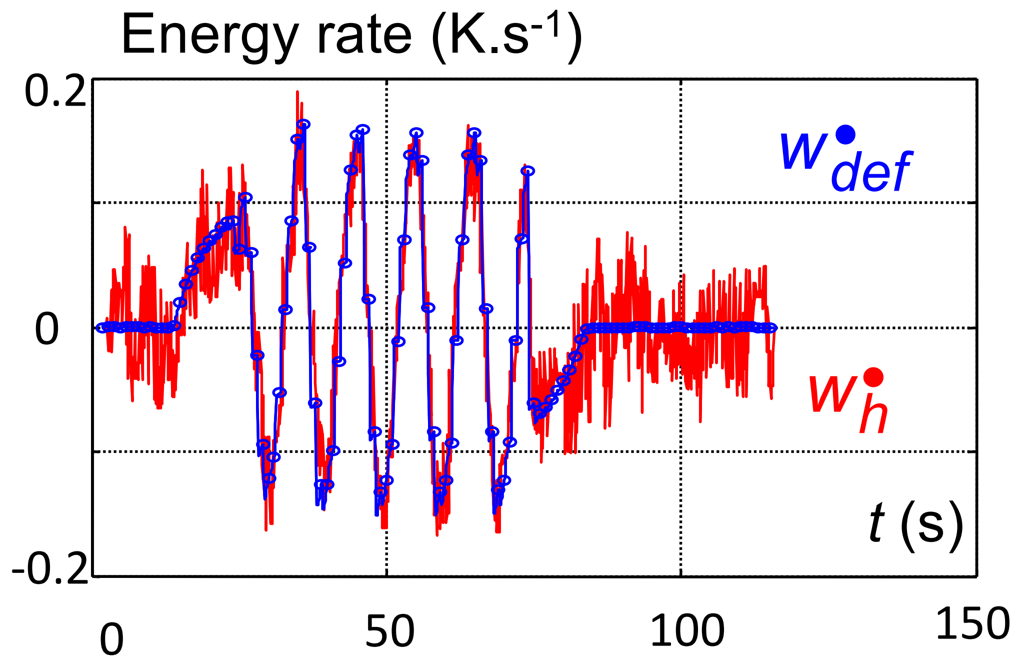


- 3 tests , 5 « internal » loops
- displacement controlled tests
- hysteresis loops
- thm couplings vs. dissipation

Energy balance at large strain

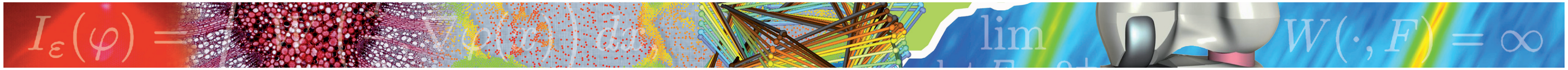
[Caborgan, PhD 11]

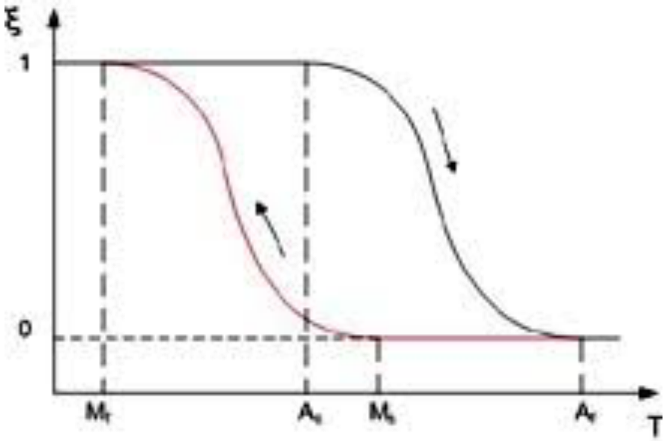




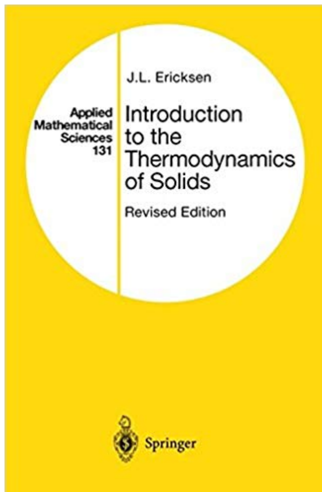
In agreement with the « perfect gaz » hypothesis

$$\dot{w}_{def} = \dot{w}_{h}$$





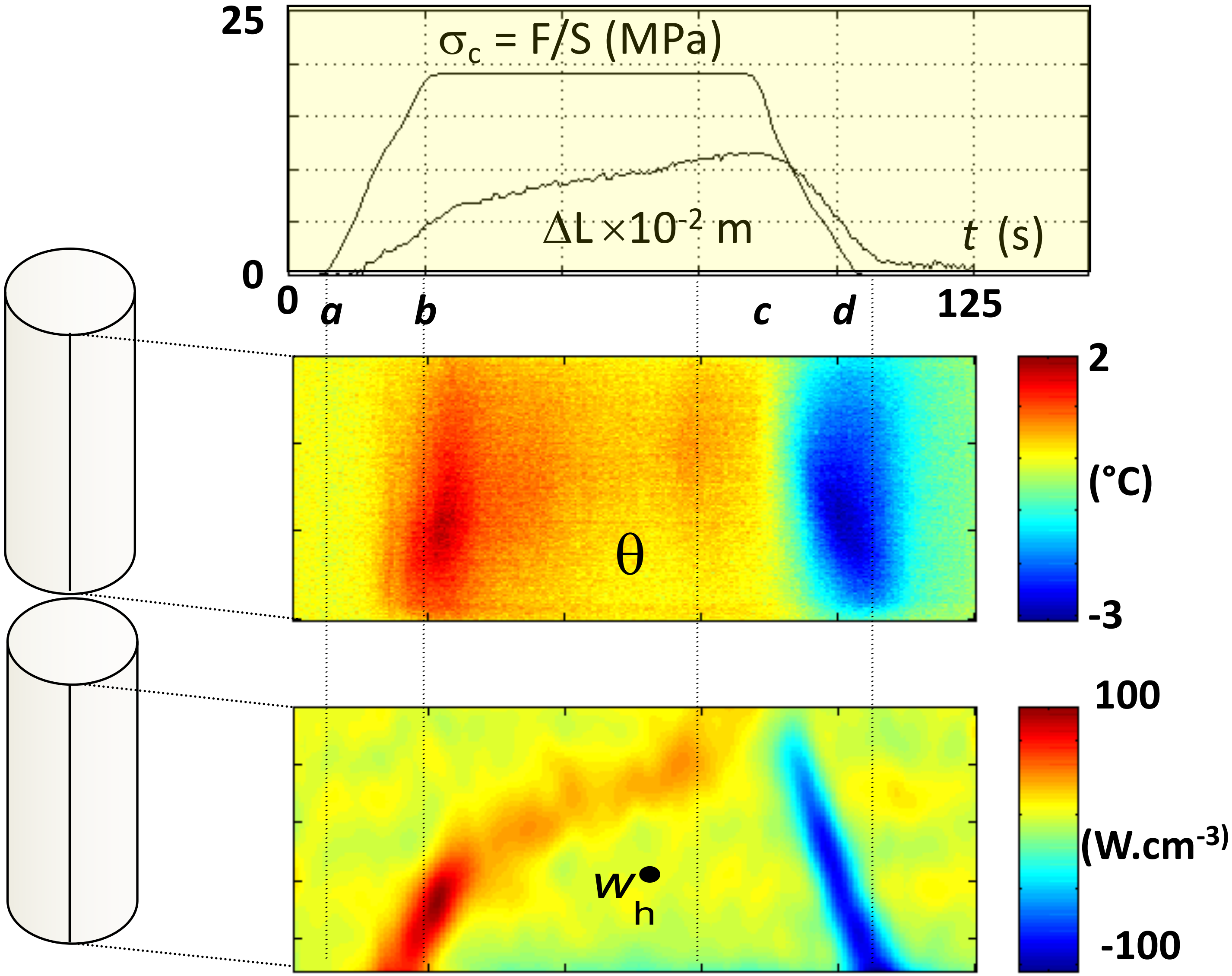
Transition de phase solide-solide
Pseudoélasticité des AMF



Pseudoélasticité des AMF : un «vieil» exemple

M. Löbel (95) → X. Balandraud (00) → S. Leclercq (01)

Monocristaux d'AMF CuZnAl CuAlBe



- fluage de l'éprouvette

- diffusion

- propagation d'un front

Energy balance for "pseudoelastic" SMA

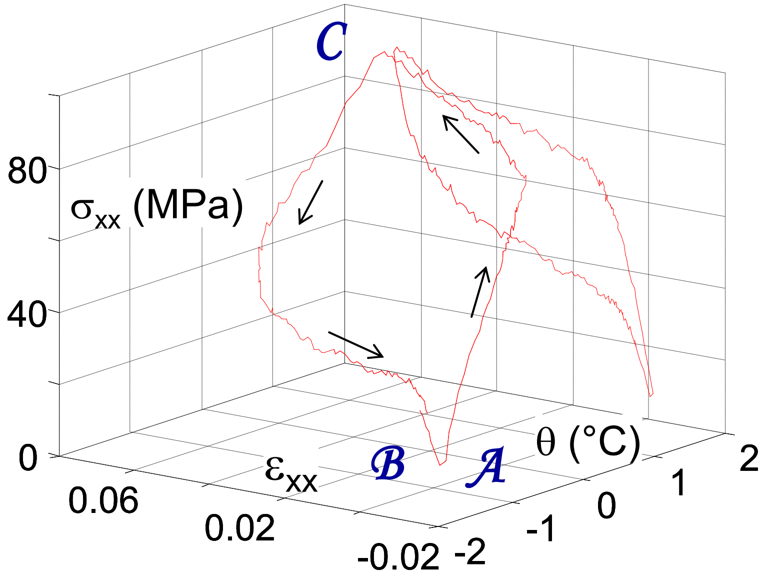
Load-unload cycle

$\mathcal{A}_h \approx 1{,}9$ MJ.m^{-3}

$w_{def}\big]_{\mathcal{A}}^{C} = 4{,}9$ MJ.m^{-3}

$w_h\big]_{\mathcal{A}}^{C} = 28$ MJ.m^{-3}

CuAlBe



specimen response (macroscale)

$$R_M = \frac{\mathcal{A}_h}{w_{def}} \approx 0{,}39$$

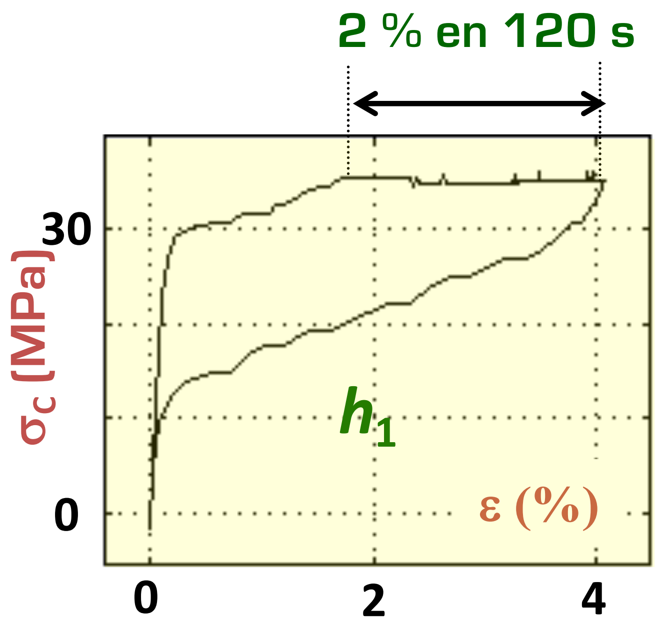
$$0 \le R_T = \frac{\int w_h \, d\tau}{\int |w_h| \, d\tau} \approx 0{,}026 \le 1$$

[S. Vigneron, PhD 09]

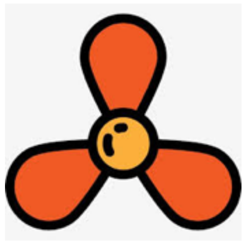
$R_T = 0$ non dissipative (intrinsically) + isothermal or adiabatic process

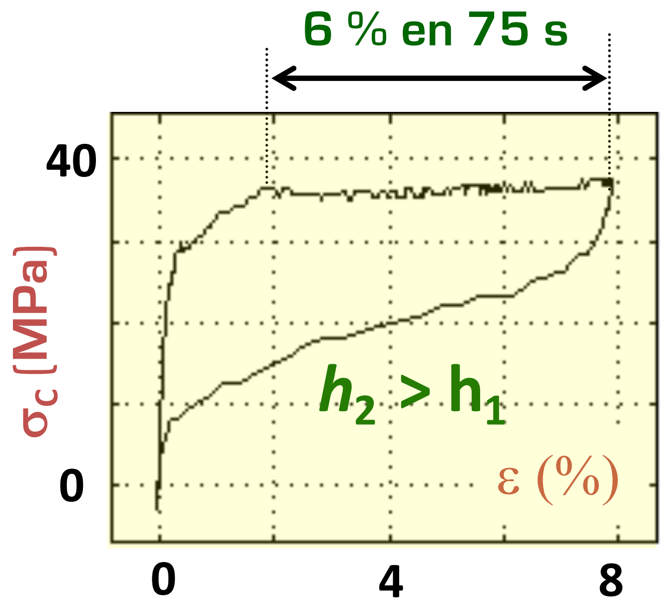
$R_T = 1$ purely dissipative (intrinsically) process without any coupling effect

strong thermomechanical couplings

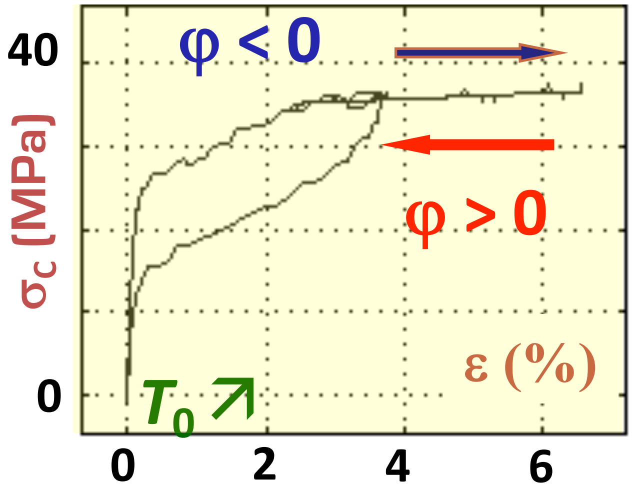


Convection naturelle





Convection faiblement turbulente



Inversion du flux

Vitesse de fluage : CL thermiques

Fluage «inverse»

Hystérésis (R_T<5 %)

Comportement ThM couplé :

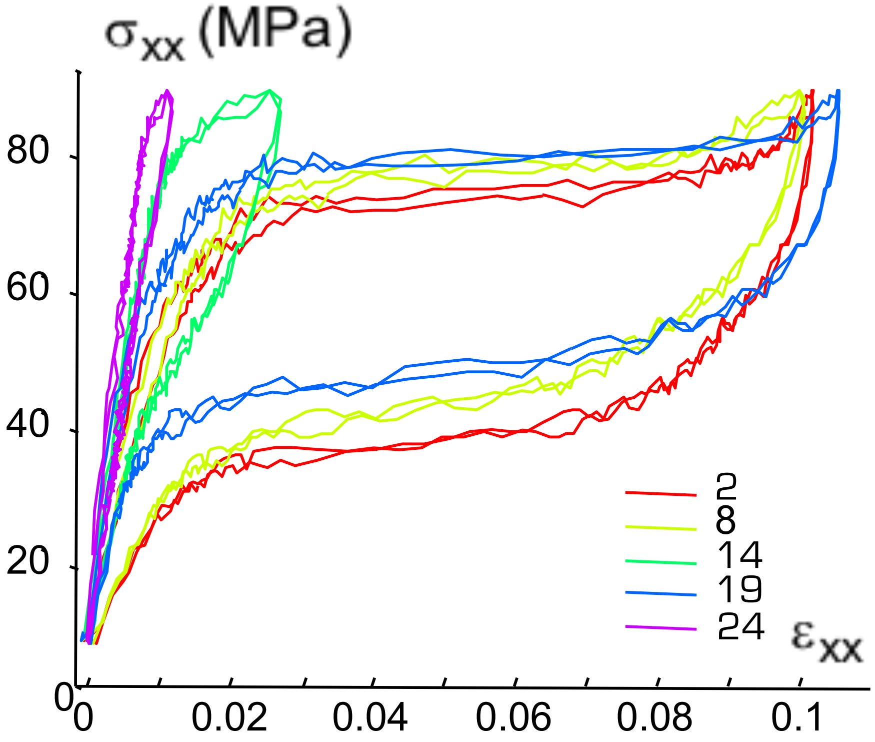
Transformation de phase anisotherme sous contrainte

Heterogeneous mesoscopic responses

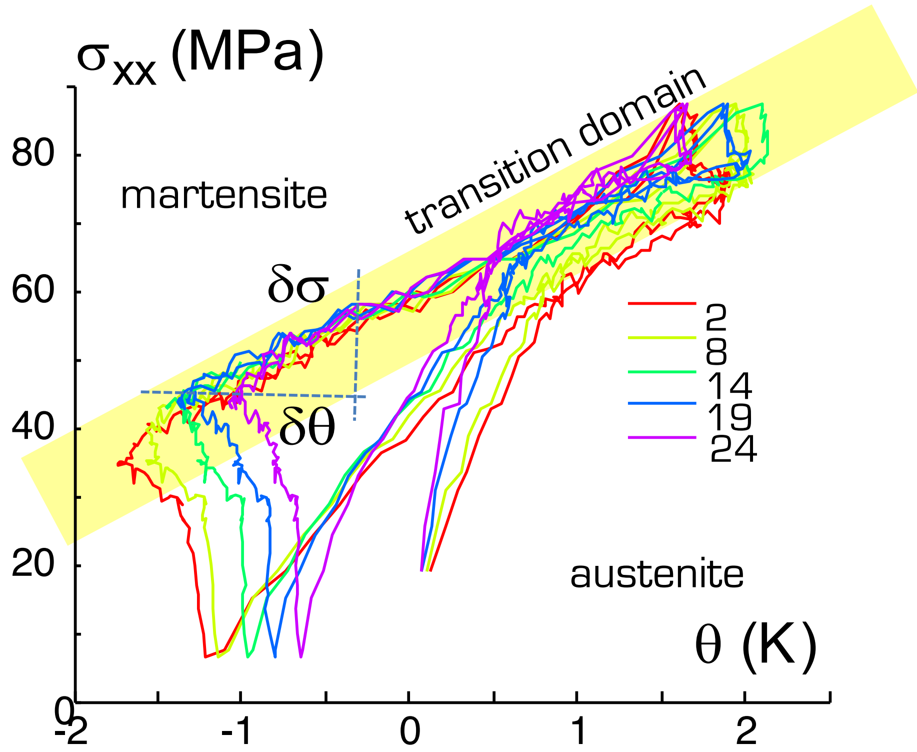
CuAlBe

meso

Localization of phase change
Propagation of phase change front



stress-strain diagram



phase diagram

$\delta\theta$ transition domain «thickness» ~1K (15 K for the SMA producer !)

Modélisation : 1ers ingrédients

- Petites perturbations thm $\theta = T - T_0 \qquad T_0$

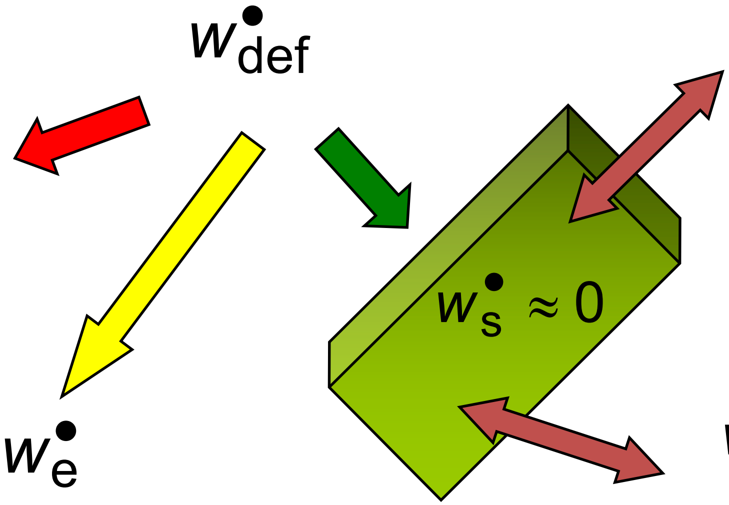
- Cinématique $\varepsilon = \varepsilon_e + \varepsilon_\theta + \varepsilon_X \qquad \boldsymbol{\varepsilon}_X(X) = \sum_n X_n\, \boldsymbol{\beta}_n$

- Énergie libre $\psi(T, \boldsymbol{\varepsilon}, X) = \psi_{the}(T, \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_X(X)) + \psi_{tr}(T, X)$

- Potentiel de dissipation $\varphi\left(\nabla T, \dot{\varepsilon}, \dot{X}\right) \Rightarrow \varphi\left(\nabla T\right)$

$$\begin{cases} \sigma^{ir} = 0 \\ Z = 0 \\ \text{si } \dot{X} \neq 0 \end{cases}$$

- Bilan d'énergie



$\overset{\bullet}{w}_{cl} = T\, Z_{,T} . \dot{X}$

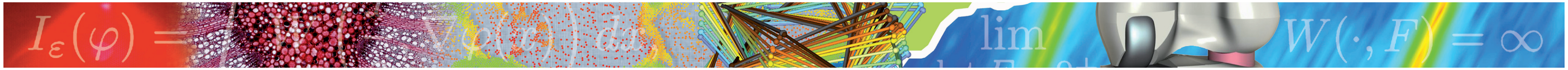
chaleur latente

$\overset{\bullet}{w}_{the} = T\, \boldsymbol{\sigma}^{r}_{,T} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_X)$

thermoélasticité

$Z = f(T, \varepsilon, X) = 0$

cinétique +
diagramme
de phase



Modèle ThM-1D

- traction

$$\sigma \geq 0$$

- quasi-staticité

$$\sigma_{,x} = 0$$

- cinématique

$$\varepsilon = \frac{\sigma}{E} + \alpha\theta + \beta X$$

- cinétique (austénite-martensite)

$$Z\dot{X} = 0$$

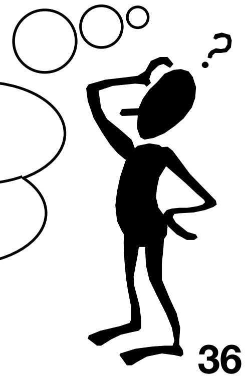
force nulle si
transformation de phase

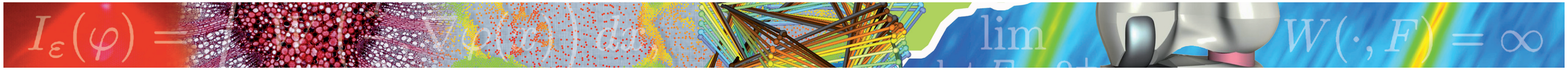
Constantes

E : Module d'Young
α : dilatation
β : transformation

k : conduction
ρ : masse volumique
C : chaleur spécifique

$$\begin{cases} n_\alpha + n_\beta = 1 \\ \mu_\alpha = \mu_\beta \\ 0 \leq X = n_\alpha - n_\beta \leq 1 \\ Z = \mu_\alpha - \mu_\beta \end{cases}$$





Modèle ThM-1D

- diagramme de phase

«Martensite» : G.B. Olson & W.S. Owen Chap. 12, 1991.

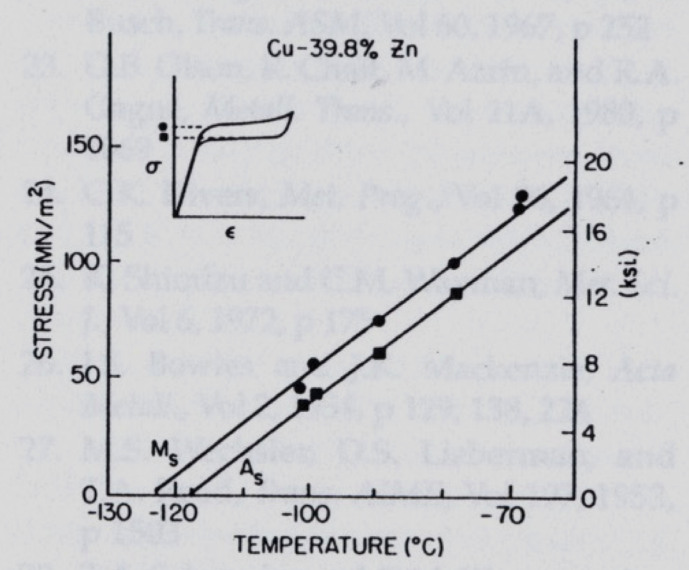
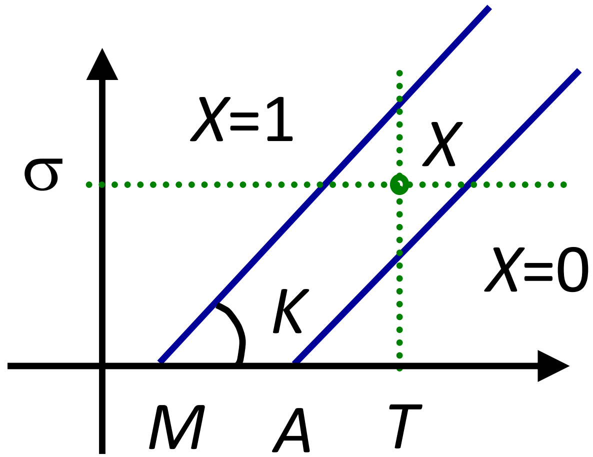


Fig. 9 Temperature dependence of stress required to produce stress-induced martensite in a Cu-Zn alloy

- Cinématique (algébrique !)

$$X = \min\left(1, \left\langle \frac{\sigma - K(T-A)}{K(A-M)} \right\rangle^{+}\right)$$



- existence d'un front

$$A - M << A \qquad A - M \approx 10^{-3} \circ C$$

i.e. localisation des déformations et des sources de chaleur

Essai numérique (I)

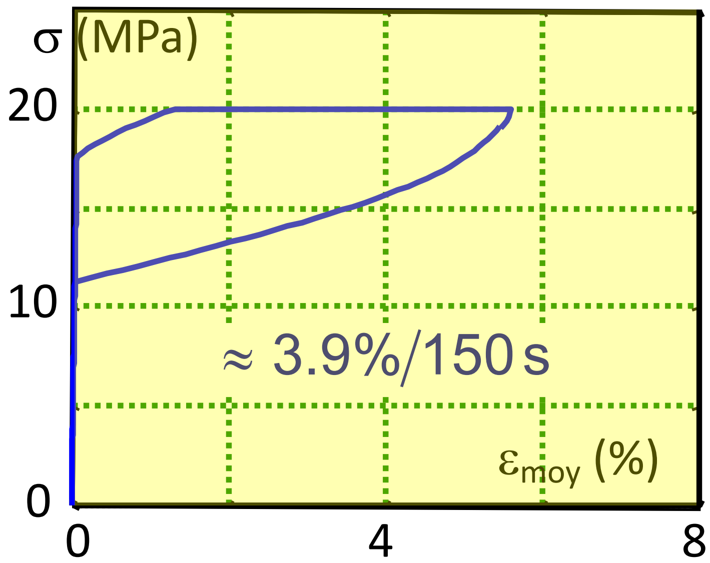
Coefficient d'échange et vitesse du front

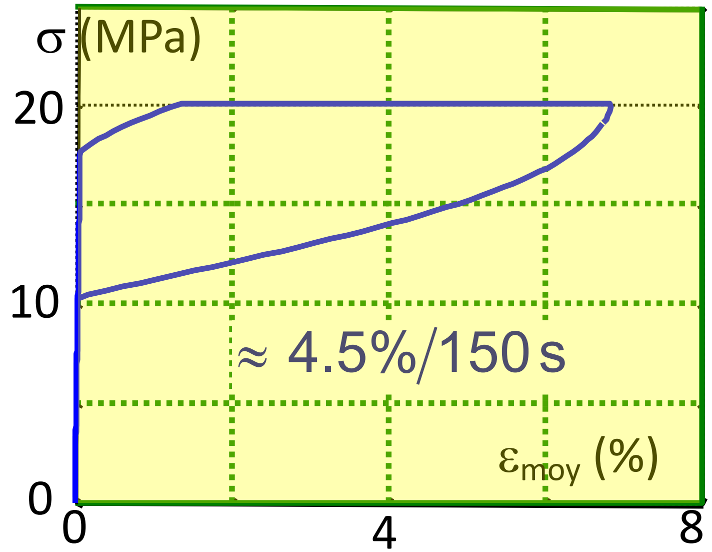
$h_1 = 10\ W/m^2K$

convection naturelle

$h_2 = 60\ W/m^2K$

légère turbulence

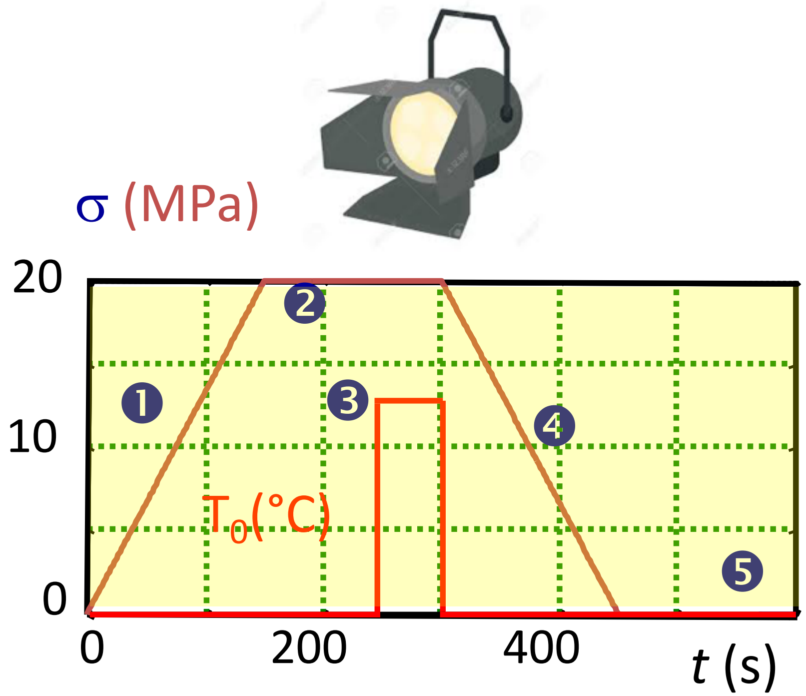




fluage + hystérésis … sans viscosité

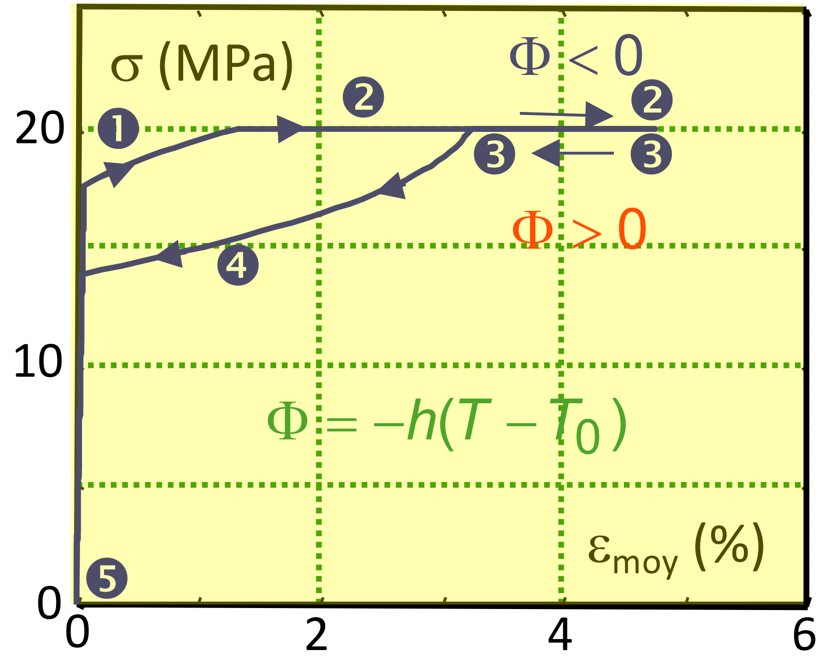
Essai numérique (II)

Fluage et fluage « inverse »



chargement thm

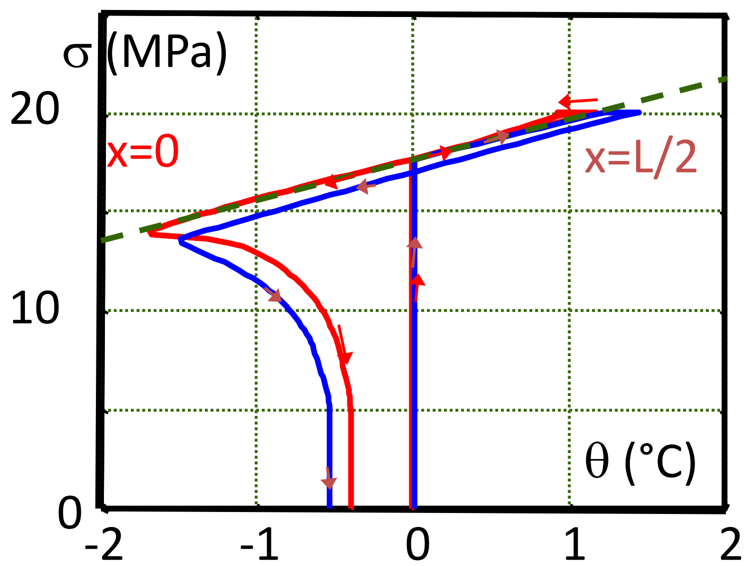
T_0 température ambiante



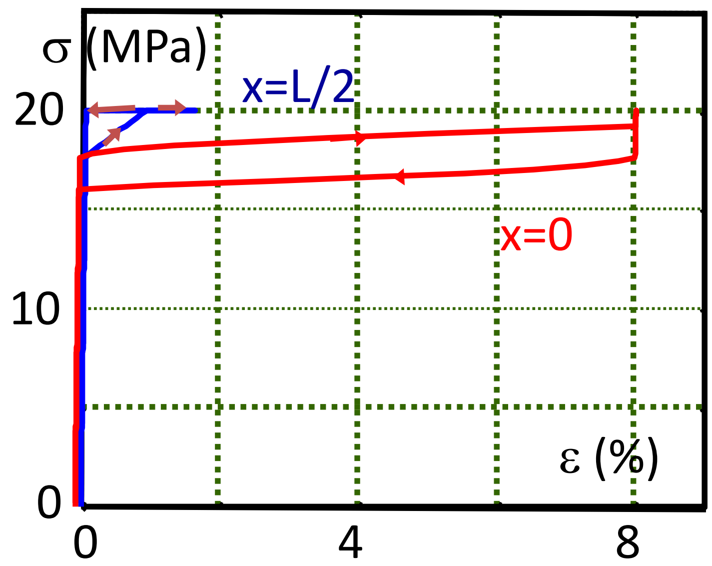
Sens du fluage induits par les échanges
de chaleur entre l'éprouvette et le milieu extérieur

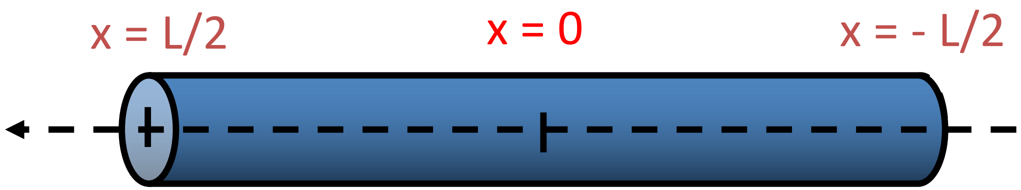
Couplage + diffusion = effet d'échelle

Trajectoires thermomécaniques



Réponses mécaniques





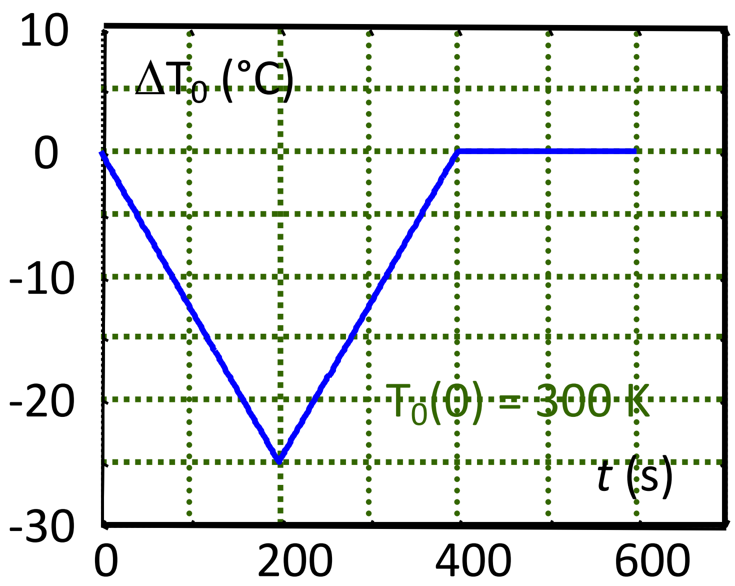
réponses locales ≠ réponse globale

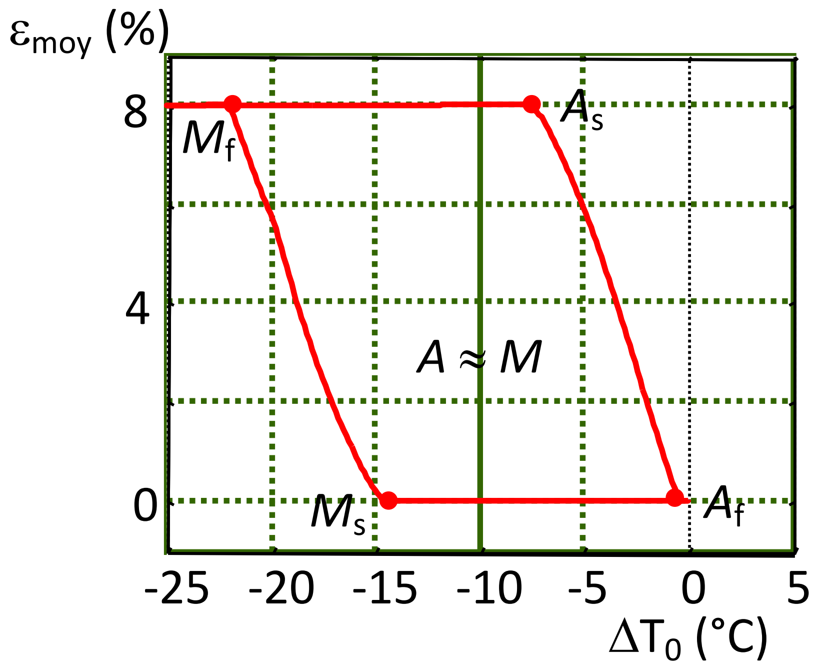
identification des équations de comportement

Essai numérique (III)

Test de dilatométrie (σ = Cte)

Domaine de transition (A - M =10^{-3}°C)



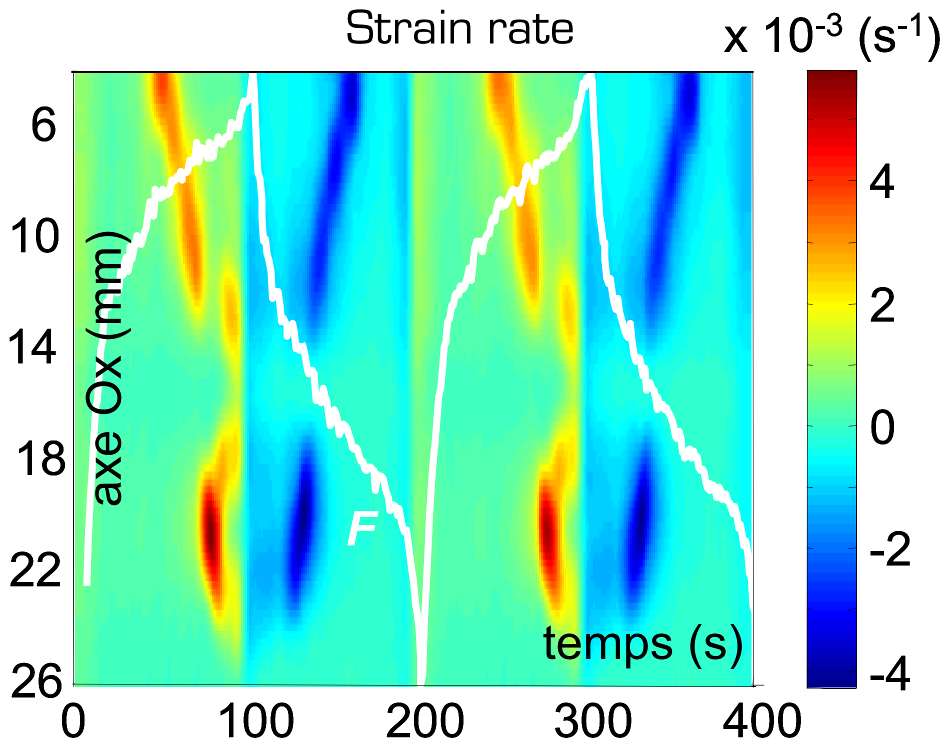


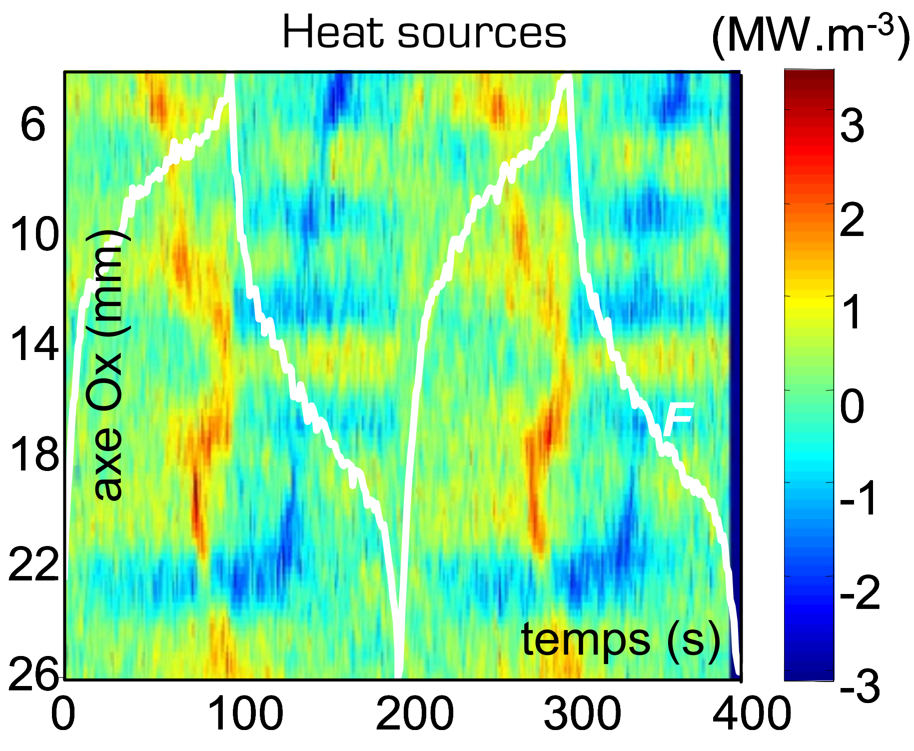
Diffusion de la chaleur, inertie thermique …

A - M = 10^{-3} °C : domaine de transition du matériau

A_s, A_f, M_s, M_f : domaine de transition de l'échantillon

Load-unload cycles

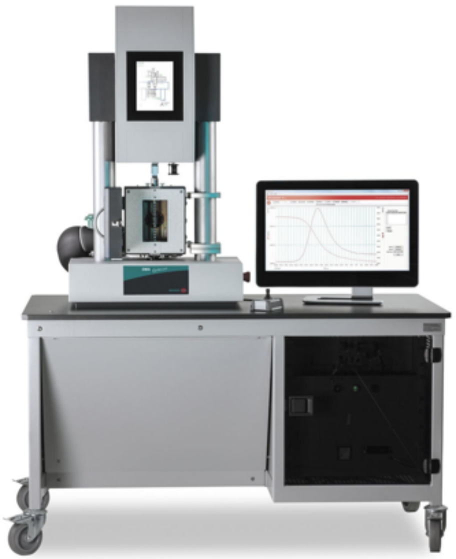




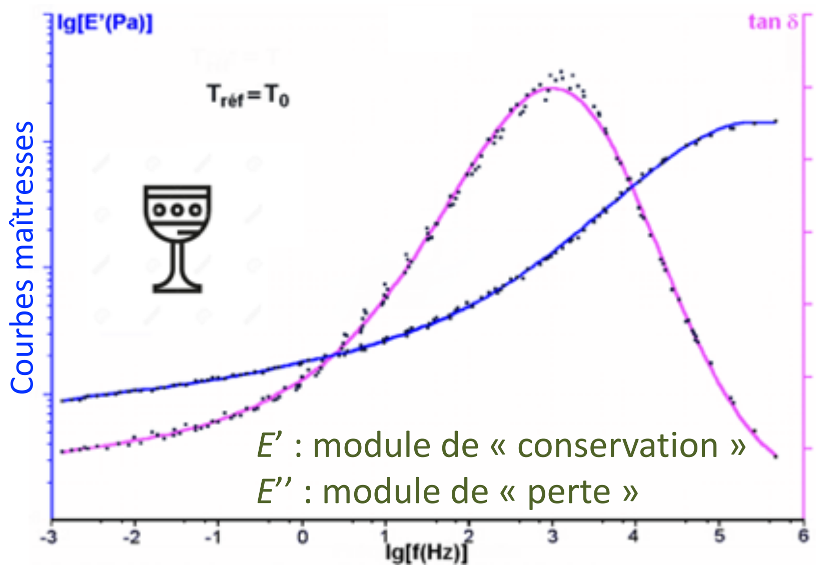
Field of local responses (mesoscale)

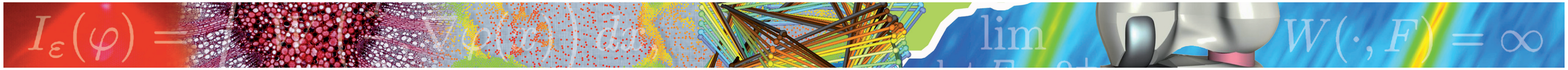
[S. Vigneron, PhD 09]

CuAlBe



Effet du temps : viscosité ou
couplage fort et dissipation thermique



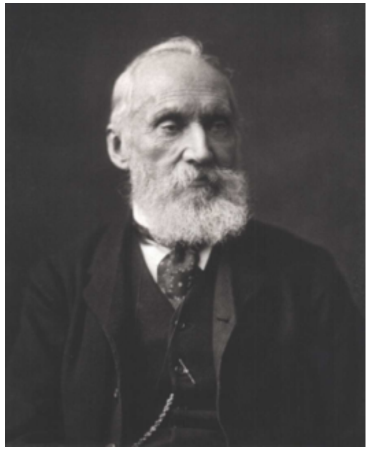


$$\begin{cases} \varepsilon = \dfrac{\sigma}{E} + \lambda_{th}(T - T_0) \\ \dot{T} + \dfrac{T - T_0}{\tau_{th}} = -\dfrac{E\lambda_{th} T \dot{\varepsilon}}{\rho C} \end{cases}$$

⇔

« 0D » approach

linear heat losses

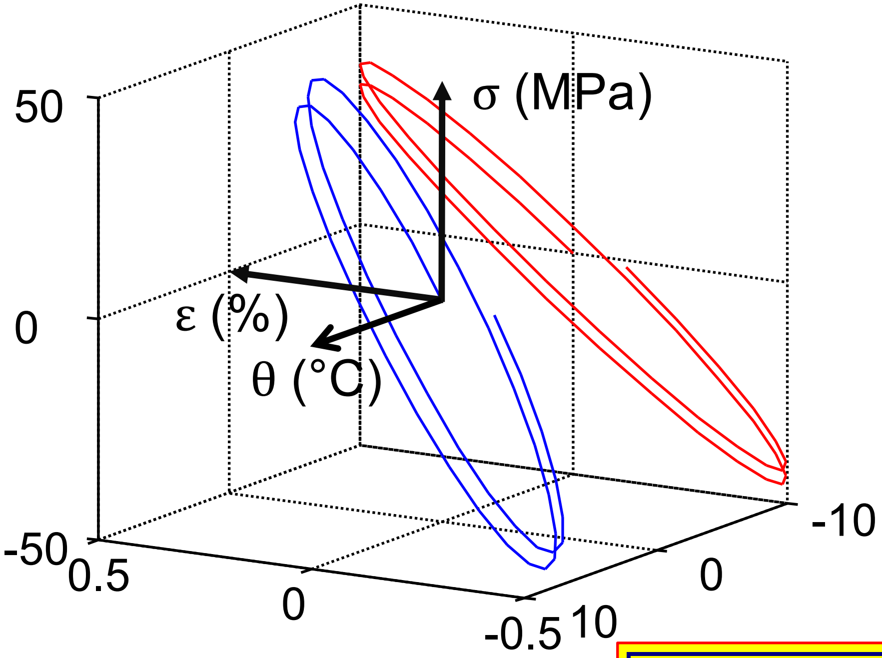


W. Thomson – Lord Kelvin

1824-1907

E = 1000 MPa
ρ = 1000 kg.m^{-3}
C = 1000 J.kg^{-1}.K^{-1}

λ_{th} = 50 10^{-5} K^{-1}×**100**
τ_{th} = 30 s
T_0 = 294 K



Toward a stabilised cycle…

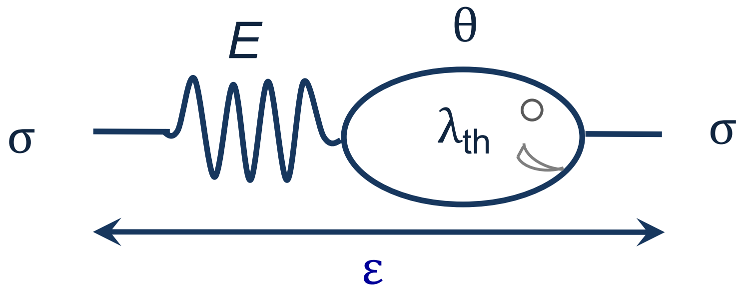
Thm couplings + thermal dissipation

$$\tilde{w}_{def} = A_h = \tilde{w}_{the}$$

Couplage ou dissipation ?

Couplage thermomélastique (i.e. d_1=0)

couplage fort



variables d'état $(\theta,\ \varepsilon)$

$$\begin{cases} \varepsilon = \dfrac{\sigma}{E} + \lambda_{th}\theta \\ \dot{\theta} + \dfrac{\theta}{\tau_{th}} = -\dfrac{E\lambda_{th}(T_0+\theta)\dot{\varepsilon}}{\rho_0 C_0} \end{cases}$$

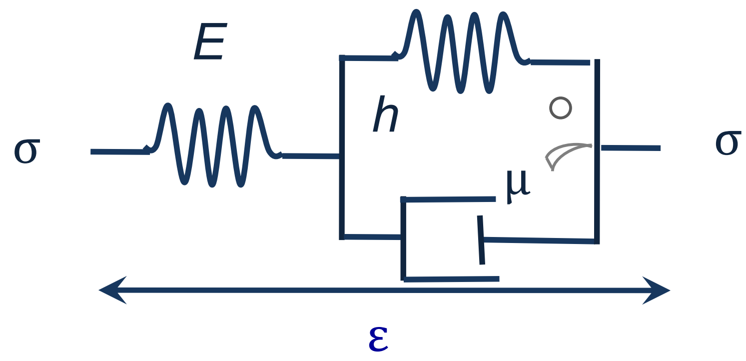
équation rhéologique

$(T/T_0 \approx 1)$

$$\sigma + \tau_{th}\,\dot{\sigma} \approx E\varepsilon + E\tau_{th}\left(1+\chi\right)\dot{\varepsilon}$$

Processus dissipatif visqueux

couplage faible



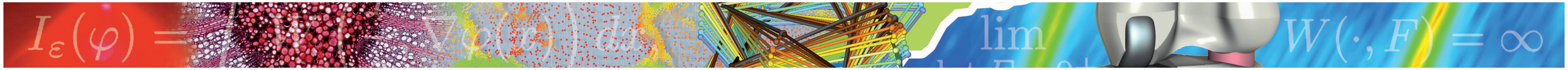
variables d'état $(\varepsilon,\ \varepsilon_v)$

$$\begin{cases} \sigma = E(\varepsilon - \varepsilon_v) \\ \sigma = h\varepsilon_v + \mu\,\dot{\varepsilon}_v \end{cases}$$

équation rhéologique

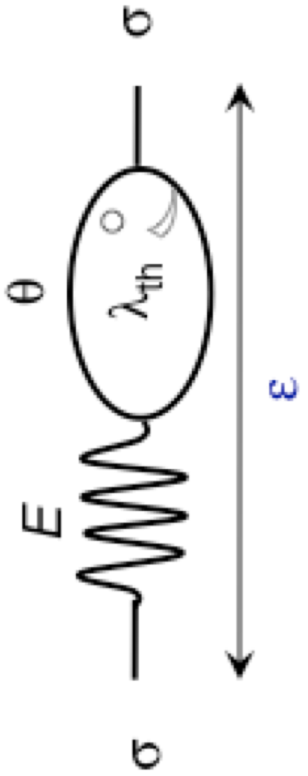
$$\sigma + \frac{\mu}{E+h}\dot{\sigma} = \frac{Eh}{E+h}\varepsilon + \frac{E\mu}{E+h}\dot{\varepsilon}$$

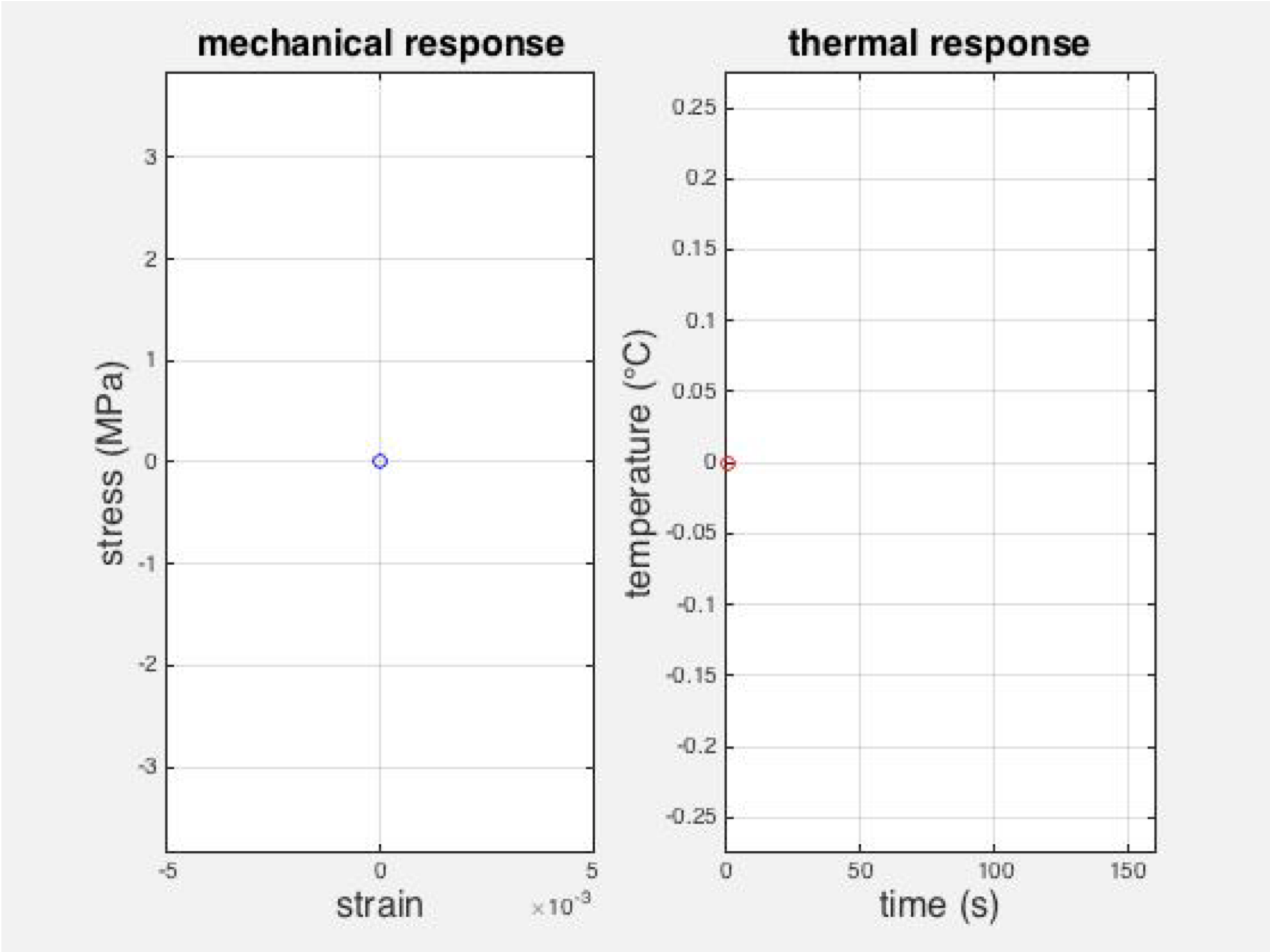
Visco-analyse de polymère (DMA)



Effets thermoélastiques : couplage fort

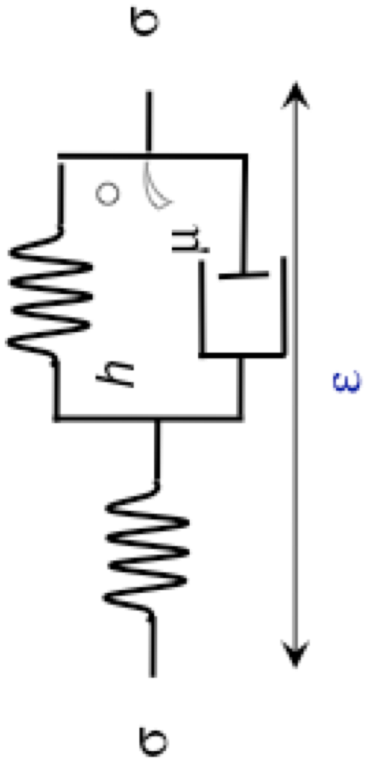
$\delta\theta$=0.5 K

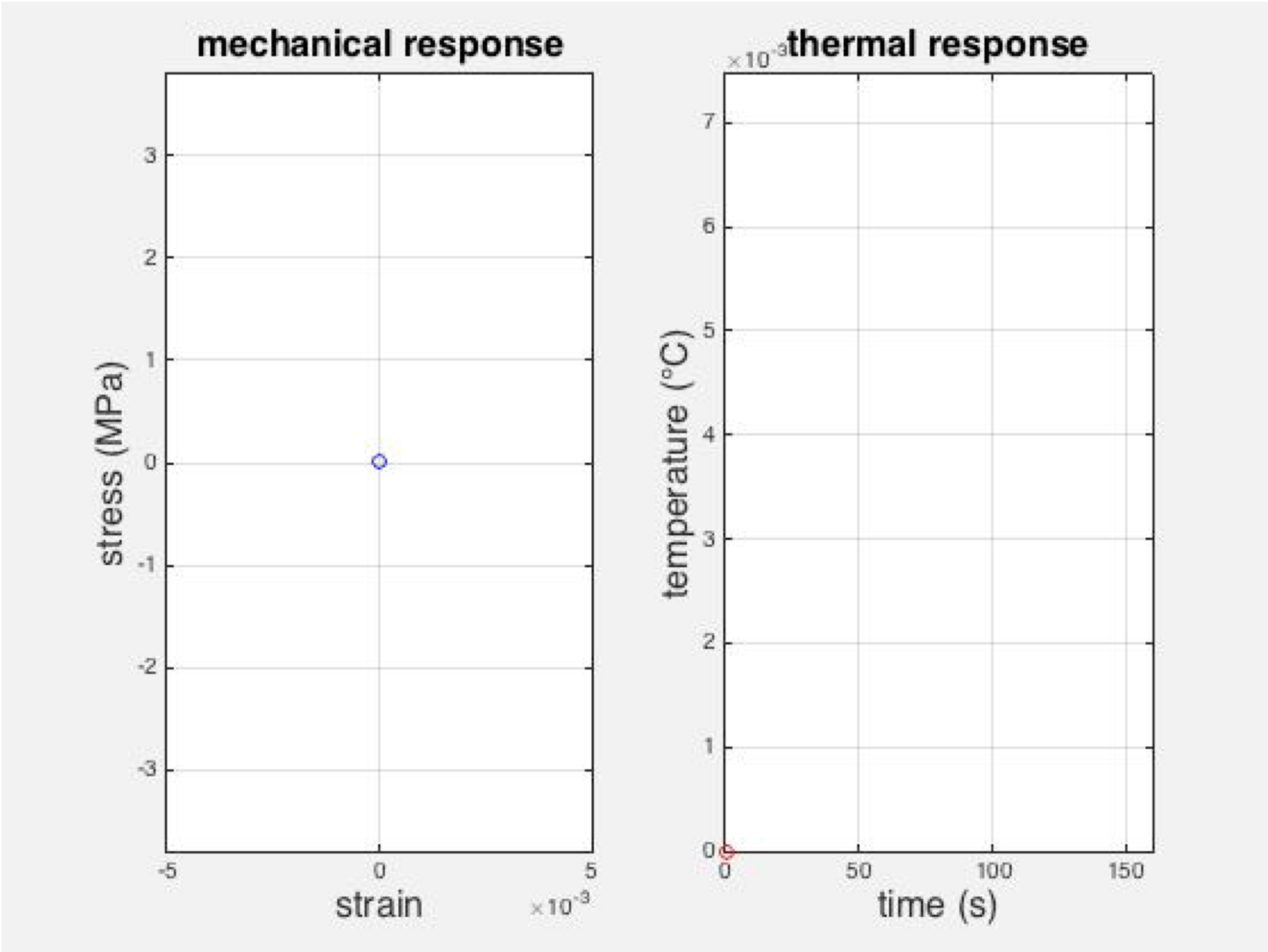




Effets visqueux : couplage faible

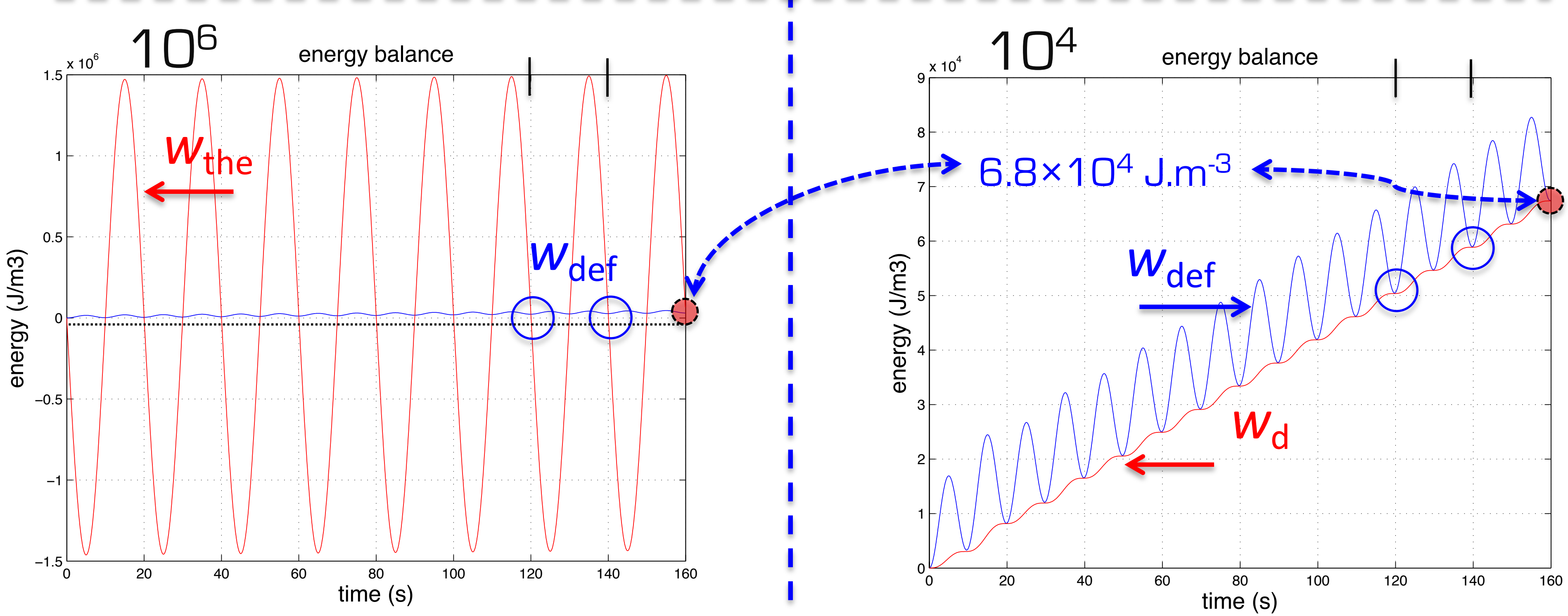
$\Delta\theta \approx 7.5\ 10^{-3}$ K





Energy balance : comparison

Thermoelasticity vs. Viscoelasticity



$\Delta w_{the} >> \Delta w_{def}$ Δ = ranges $w_d \leq w_{def}$

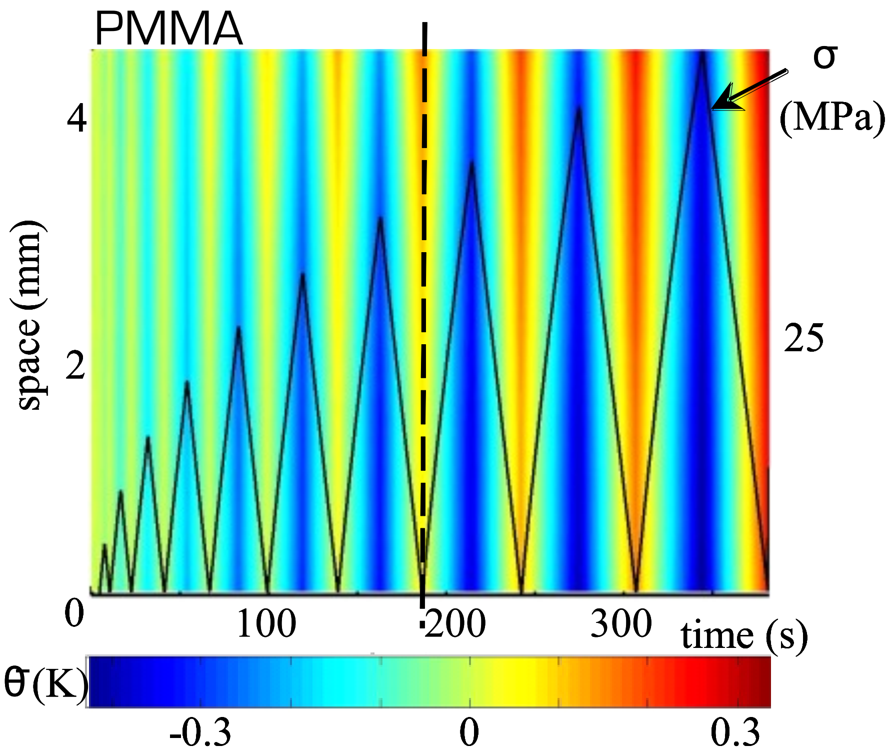
$w_{the} = w_{def}$ ○ = end of cycle $w_d = w_{def}$

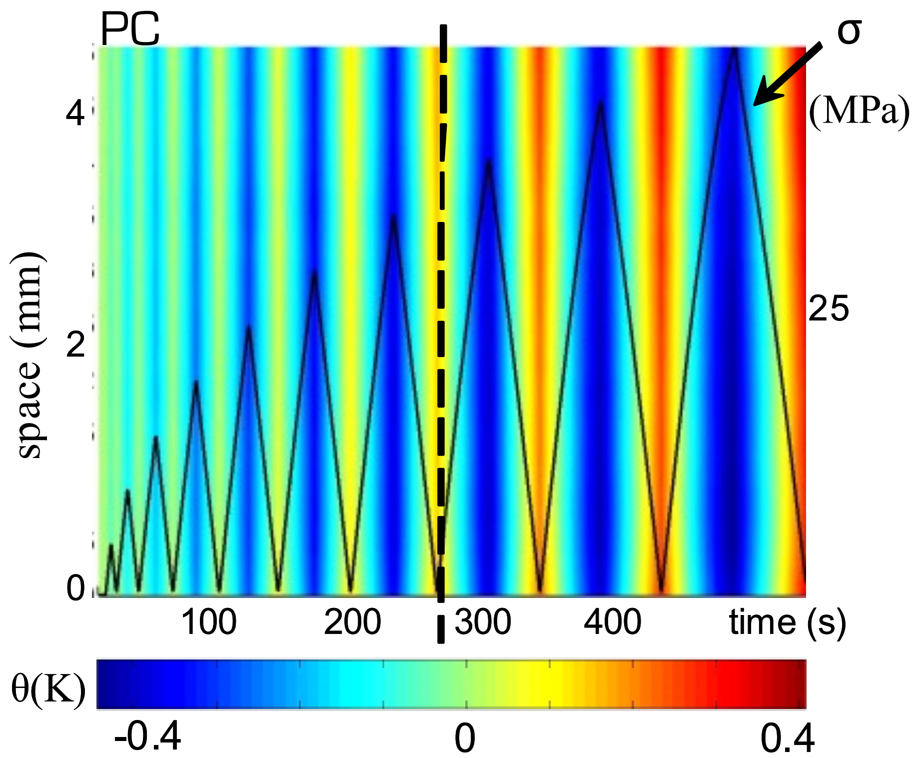
PMMA & PC : polymères à l'état vitreux

HPP isotherme : le royaume de la viscoélasticité linéaire

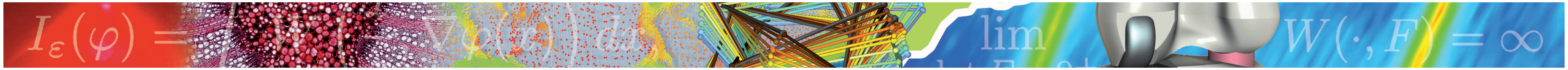
J. Alfrey (48), M. Biot (65), F. Sidoroff (70-75)

Equivalence of series and parallel models (e.g. P.T. and Z. models)



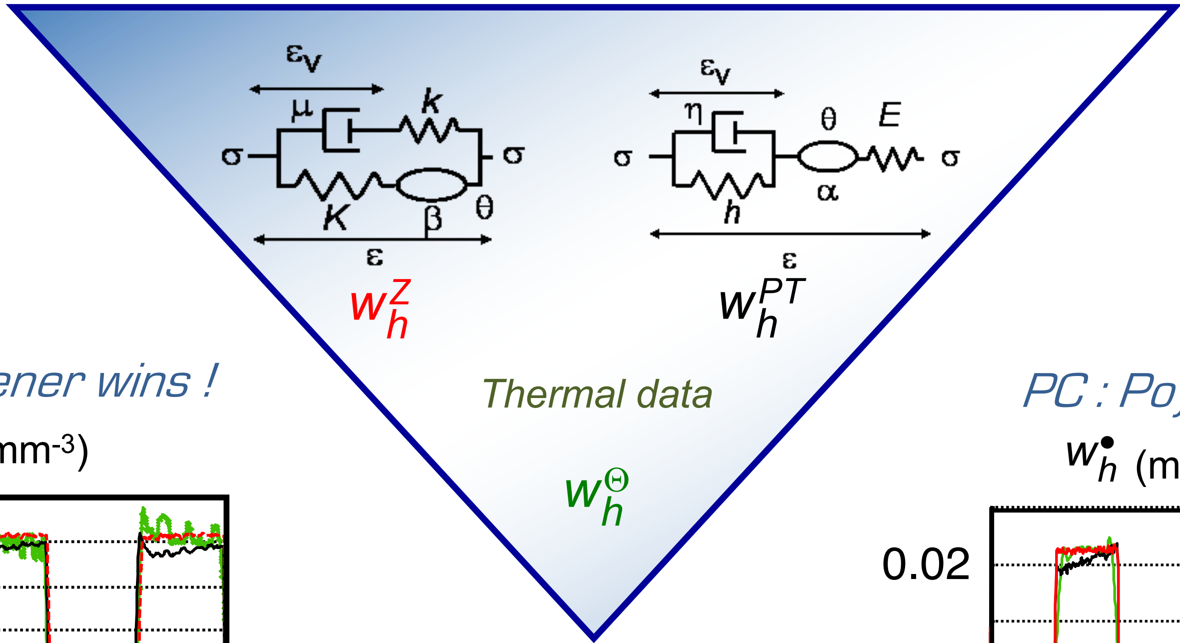


[S. Moreau, PhD 03]

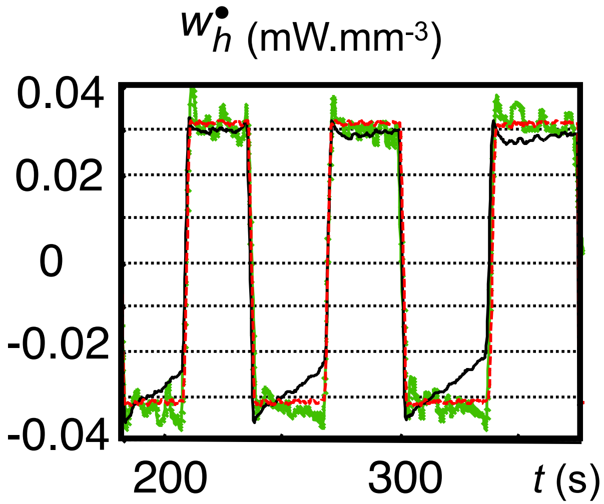


PMMA and PC : simple models for glassy polymers

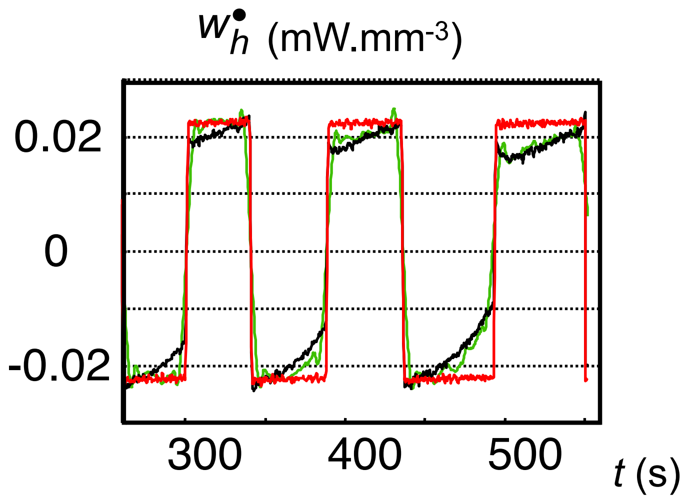
[PhD, Moreau S., CRAS 2005]



PMMA : Zener wins !



PC : Poynting wins !



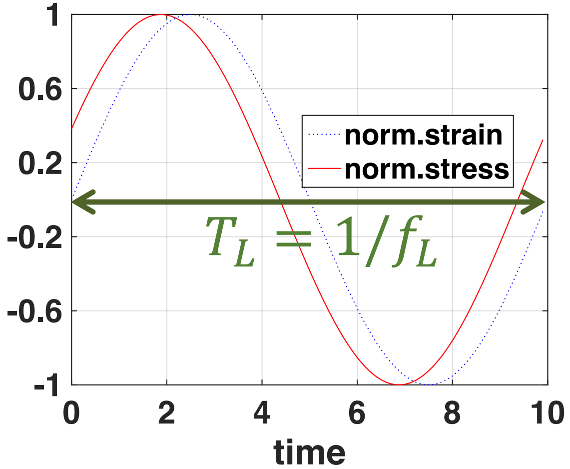
$A_h \approx w_{the}$?

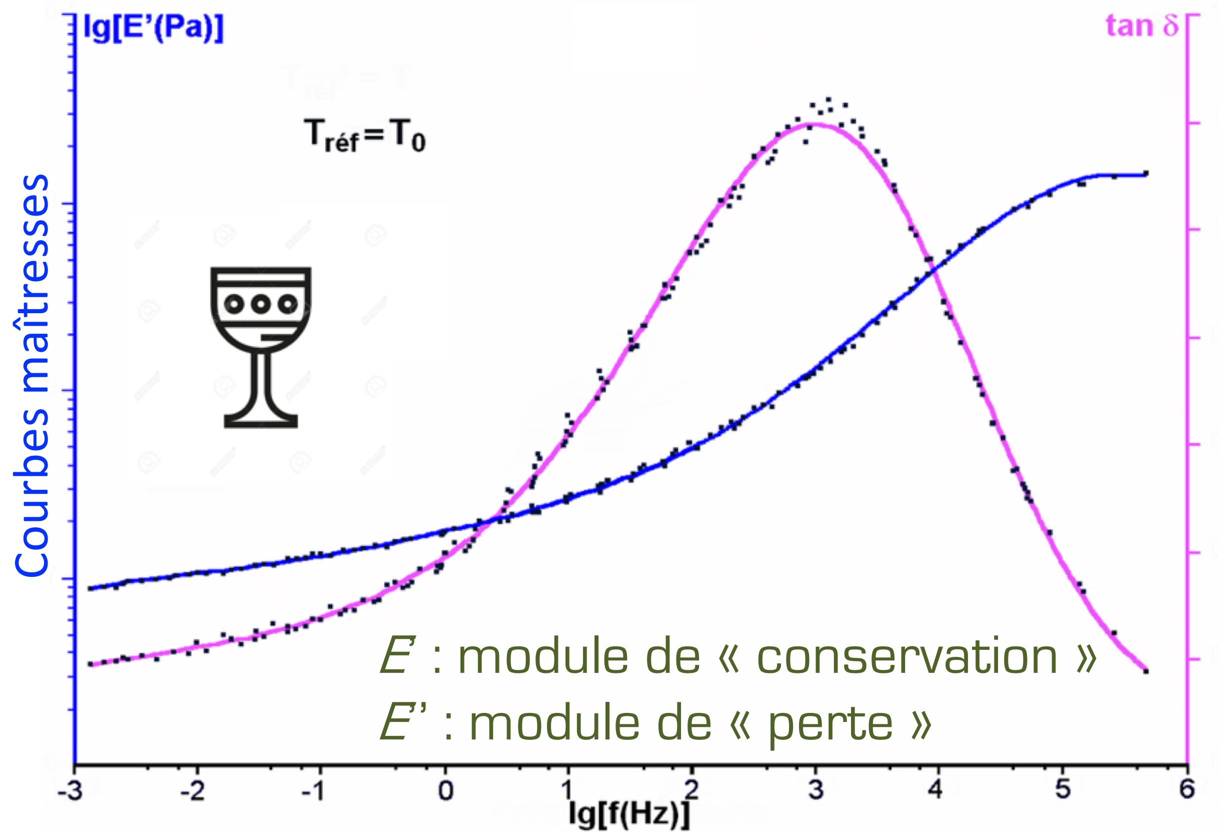
Low dissipative effects ! energy storage ?

Equivalence of series and parallel models ?

Stabilisation cyclique : polymères

Viscoélasticité linéaire : équivalence temps température (TTS - DMA)
Facteur de translation $a_{T_0}^T$





$$\varepsilon = \varepsilon_0 \sin(2\pi f_L t)$$

$$\sigma = \sigma_0 \sin(2\pi f_L t + \varphi)$$

$$\sigma = E' \varepsilon_0 \sin(2\pi f_L t) + E'' \varepsilon_0 \cos(2\pi f_L t)$$

$$\tan(\delta) = \frac{E''}{E'}$$ → Pertes Enr. Diss. ←

$$E'(f_L, T) = E'\left(f_L / a_{T_0}^T, T_0\right)$$

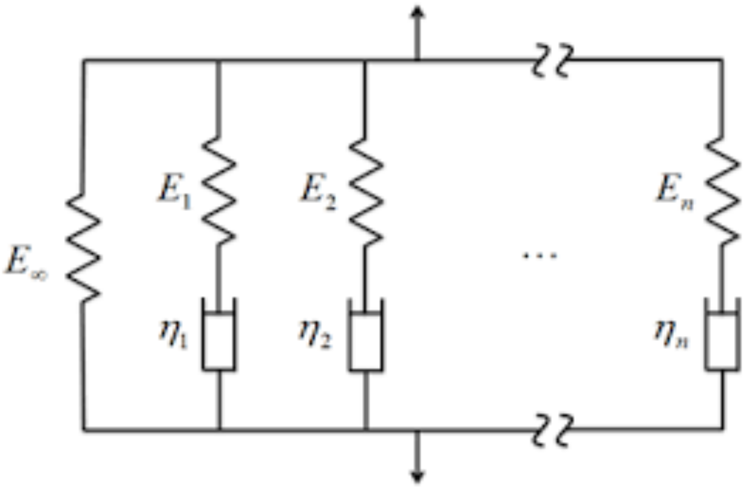
$$E''(f_L, T) = E''\left(f_L / a_{T_0}^T, T_0\right)$$

Analyse dynamique mécanique (TTS)

Idée : basse température + basse vitesse
haute température + haute vitesse

« = » même mobilité moléculaire

Modèle de Maxwell généralisé
« anisotherme»



température = paramètre du modèle ...

$\tau_i = \mu_i / E_i = \tau_i^0 \, f(T)$

Quelques ingrédients théoriques :

$$E' = E_\infty + \sum_1^N \frac{E_i \tau_i^2 \omega^2}{1 + \tau_i^2 \omega^2}$$

$$E'' = \sum_1^N \frac{E_i \tau_i \omega}{1 + \tau_i^2 \omega^2}$$

Arrhenius (ou WLF ou ...)

$$\tau_i = \tau_i^0 \exp\left(-\frac{E_a}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right)$$

La température est
une variable contrôlée ($T = T_{DMA}$)

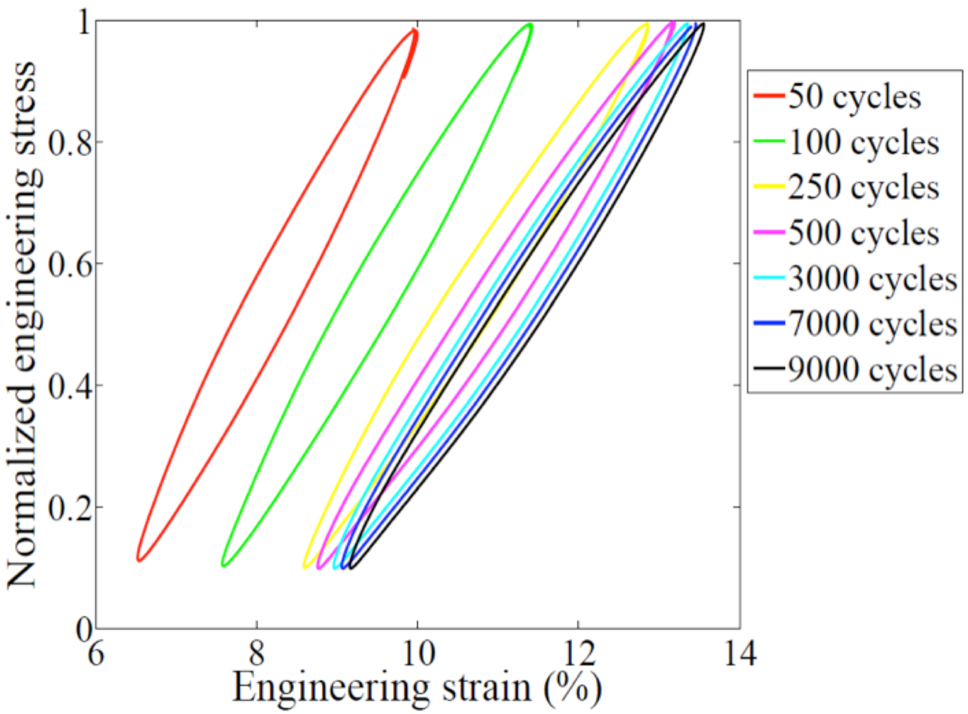
Pas de couplage, pas d'auto-échauffement !

Wet polyamide 6.6 (I)

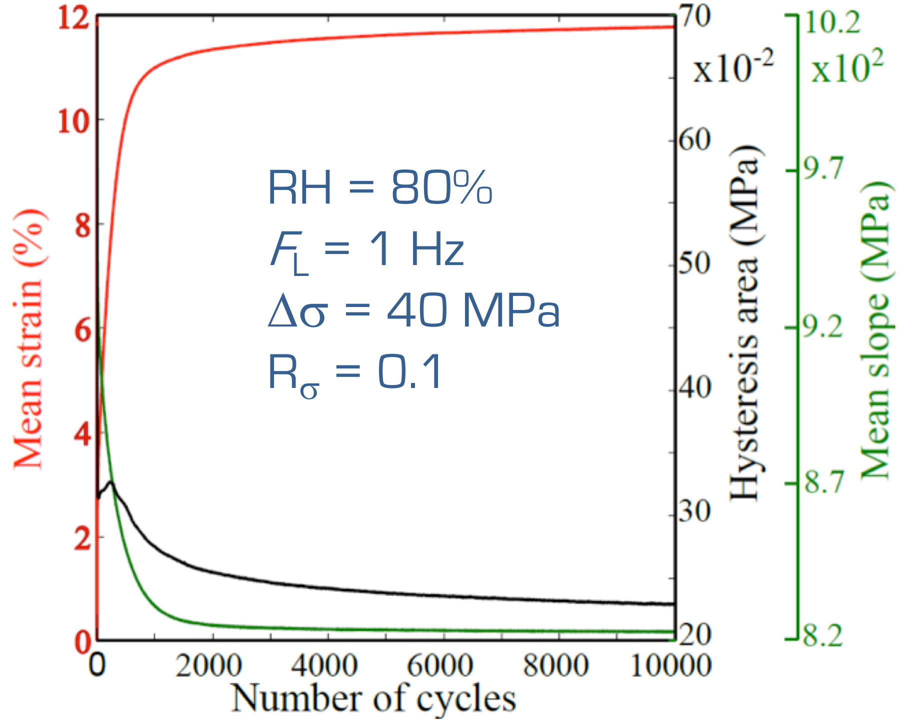
[A. Benaarbia et al., MoM, 16]

Overall mechanical responses

Stress-strain loops



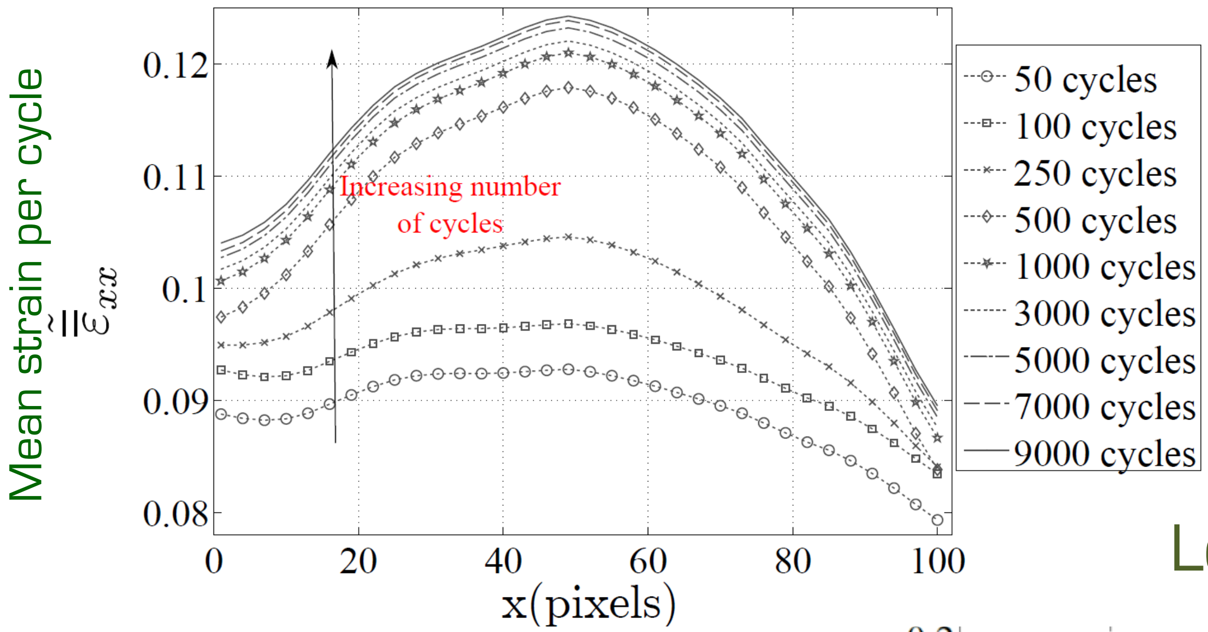
Loop parameters



- ✓ Strong initial changes
- ✓ Towards a gradual cyclic stabilization ?

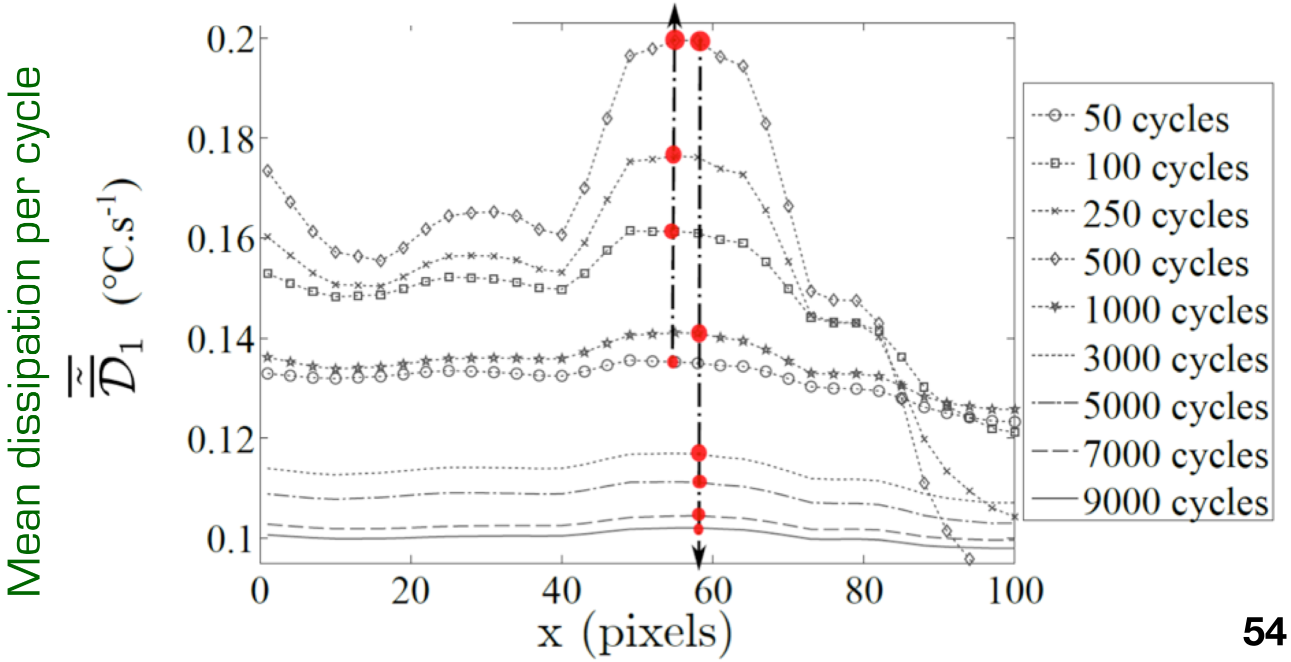
SOLVAY

Wet polyamide 6.6 (II)



Strain concentration
[0,500]
Progressive stabilization
[1000,9000]

Longitudinal profiles



Localization of dissipation
[0,500]
Progressive stabilization
[1000,9000]

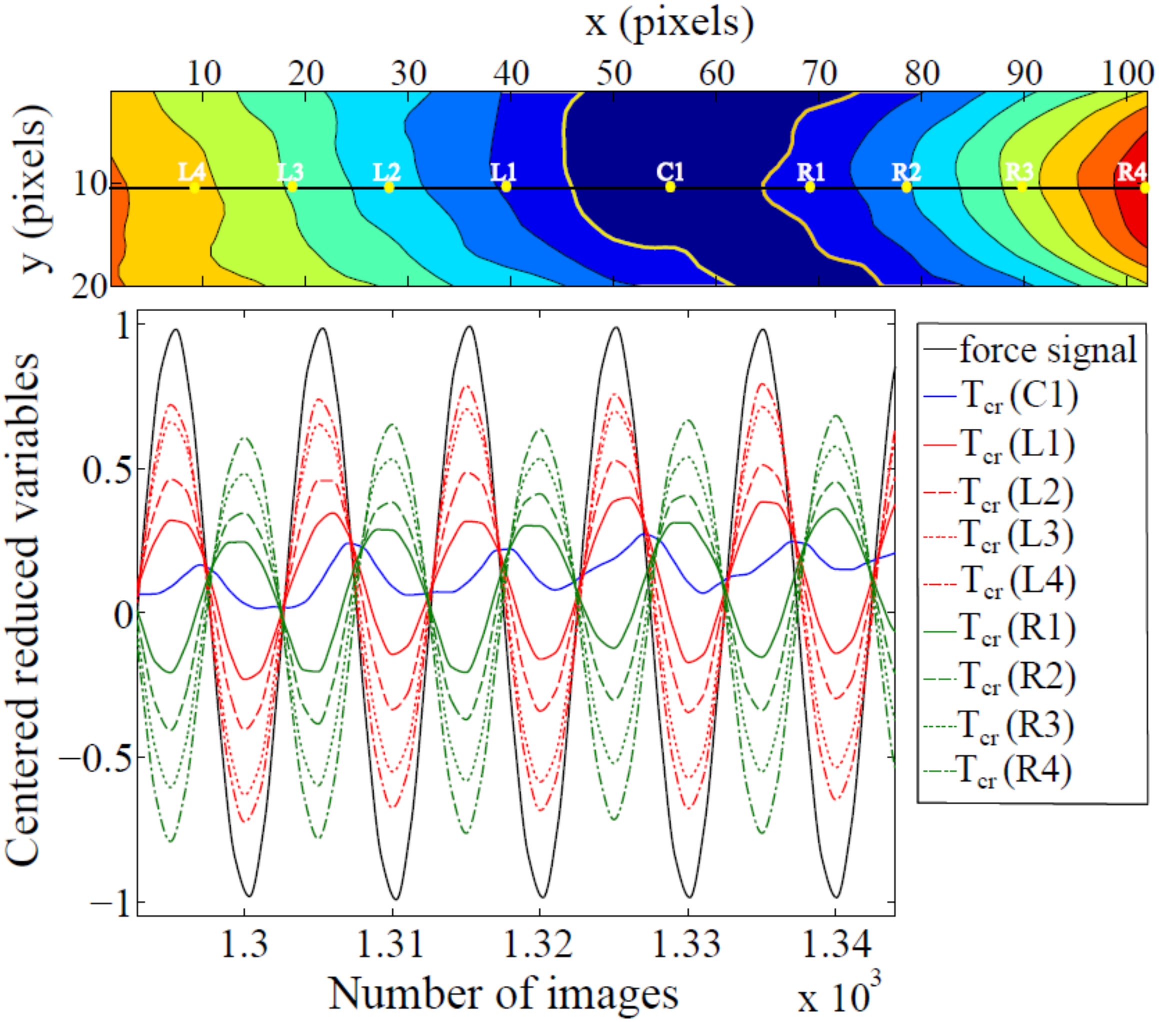
SOLVAY

Wet polyamide 6.6 (III)

[A. Benaarbia et al., MoM, 16]

Local "thermoelastic" temperature ranges

RH = 80%
F_L = 1 Hz
R_s= 0.1
Ds = 40 MPa
Cycle #100



Rubber elas.

Standard elas.

Wet polyamide 6.6 (IV)

Comments

- ✓ **Self heating** induced by dissipation (20 °C)
- ✓ Glass transition temperature Tg (close to RT !)
- ✓ From glassy material to rubbery material ??
- ✓ **Water plasticizing** (Tg $\searrow$ when RH$\nearrow$)
- ✓ Chemical ageing
- ✓ Thermo-hygro-chemio-mechanical modeling
- ✓ **Role of glass fibers** … multiscale approach
- ✓ Structures designed for **fatigue**